Wave-Number Selection and Phase Solitons in Spatially Forced Temporal Mixing Layers

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The temporally growing shear layer produced by tilting a tube filled with two immiscible fluids of different density is spatially forced via modulations of one of the lateral boundaries. The competition between the forcing wave number k_f and the intrinsic wave number k_n leads to a mode diagram that is compatible with corresponding observations of spatial mixing layers. As the detuning is gradually increased, the interface displays stationary phase solitons that are indicative of the existence of incommensurate states.

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In order to study pattern selection in unstable flows it is often convenient to examine the response of the system to an imposed perturbation. The effects of *spatial* forcing at a given wavelength have been investigated in the context of convective instabilities in nematic fluids by Lowe and Gollub [1]. The competition between the natural wavelength of the system and the externally applied wavelength then leads to a commensurate-incommensurate transition which is signaled by the appearance of phase solitons as shown by Coullet [2]. By contrast, in spatially developing open flows, *temporal* forcing is found to be very effective in controlling the downstream evolution of the vortical structures. For instance, spatially developing mixing layers formed by the merging of two streams of different velocity on either side of a splitter plate are extremely sensitive to an imposed temporally periodic excitation, as first demonstrated by Ho and Huang [3,4]. In this case, variations in the applied frequency give rise to frequency-locked regimes whereby the initial response frequency of the shear layer becomes an integer multiple of the forcing frequency. Furthermore, very low excitation levels can dramatically alter the downstream development of Kelvin-Helmholtz vortices and the spreading rate of the entire flow.

In the present investigation, we choose instead to examine the dynamics of *temporally evolving* mixing layers subjected to a *spatially periodic* excitation. In other words, the geometry is closed, as in convection experiments, and the instability mechanism is shear induced as in conventional spatial mixing layers. The basic experimental setup is the same as in the earlier studies of Thorpe [5,6]: A long tank filled with two immiscible fluids of different density is tilted at a finite angle from its initially horizontal position. A spatially uniform shear is produced under the action of gravity, which leads to the formation of Kelvin-Helmholtz billows. In our case, the flow is forced by periodically modulating the lateral boundaries along the stream. The main goal of the study is then to determine the response of the flow resulting from the competition between the natural wavelength associated with the Kelvin-Helmholtz instability and the externally imposed wavelength. The present observations indicate that the mode diagram of the flow exhibits the same locked regimes (commensurate states) as in spatial mixing layers. However, a closer investigation reveals the existence of incommensurate states characterized by the presence of phase solitons when the forcing wave number is slightly above the natural wave number. To our knowledge, these features extending over several characteristic wavelengths had not previously been observed in unstable shear flows.

The experimental setup is sketched in Fig. 1. A cylindrical glass tube 1.5 m long and 6 cm in diameter is filled with two immiscible fluids: salt water of density $\rho_1 = 1.018$ g/cm^3 and 1,2,3,4-tetrahydronaphtalene of density $\rho_2 = 0.97$ g/cm^3, the surface tension between the two fluids being 37 mN/m. Both fluid layers are chosen to be of equal depth in order to obtain stationary instability waves. When the tube is suddenly tilted at an angle α, typically less than 20°, an accelerating shear flow is created in the center of the apparatus, with the following density and streamwise velocity distributions:

$$\rho = \rho_1, \quad \mathbf{V}_1 = U(t)\mathbf{e}_x, \quad z < 0,$$

$$\rho = \rho_2, \quad \mathbf{V}_2 = -U(t)\mathbf{e}_x, \quad z > 0,$$

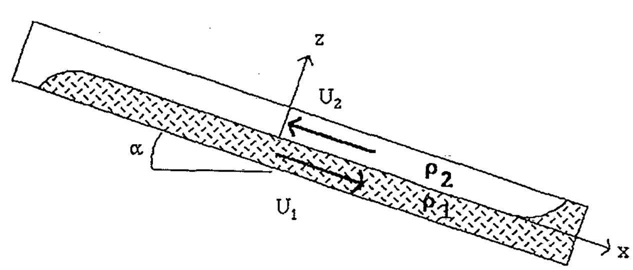

FIG. 1. Thorpe's experiment: A tank is filled with two fluids of density ρ_1 and ρ_2 and is inclined at an angle α to produce an accelerating shear flow in the center of the tube.

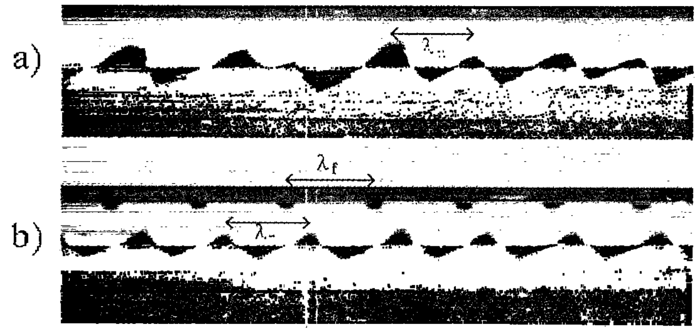

FIG. 2. Interface deformation 0.3 sec after the onset of instability, $\alpha=12.5°$, $\lambda_n=4$ cm. (a) Unforced configuration. (b) Forced configuration $\lambda_f=\lambda_n=\lambda_r$; note the presence of a modulated top boundary.

where

$$U(t)\equiv\frac{\rho_1-\rho_2}{\rho_1+\rho_2}g\sin\alpha t\,.$$

The velocity difference $V_1-V_2=2U(t)$ must exceed the critical value $\Delta U_c=0.13$ m/s for the basic flow to become unstable. One then observes growing waves on the interface, of characteristic wavelength λ_n, which can be visualized by lighting the system from behind with a neon light (Fig. 2). The evolution of the interface deformation is recorded and the images are digitized in order to interpret the data. Periodic spatial forcing of wavelength λ_f may be applied by regularly placing 3-mm-high obstacles on the upper wall and maintaining them with magnets. The duration of the experiment is typically 1.5 sec for an angle $\alpha=10°$. It is limited by the fact that hydraulic jumps move from the ends of the tank toward the center, as soon as the tube is tilted. It is also important to bear in mind that the evolution of the observed structures takes place over inertial time scales associated with the growth of the instability that are much shorter than viscous diffusion time scales. The flow can therefore be considered as purely inviscid in character, in contrast with convection experiments where dissipation is essential.

One of the obvious effects of spatial forcing is illustrated in Fig. 2. When the flow is unforced [Fig. 2(a)], the observed structures are irregularly spaced and of irregular height. If one neglects end effects, the system is invariant under continuous translations $x\rightarrow x+\text{const}$. There is no phase reference, and the development of the instability leads to the formation of a nearly periodic pattern. A natural wavelength λ_n can be defined as the average distance between neighboring wave crests over the extent of the apparatus. In the forced case [Fig. 2(b)], the flow is invariant under discrete translations $x\rightarrow x+n\lambda_f$, where λ_f is the forcing wavelength. By analogy with the work of Ho and Huang [3], one can define the response wavelength λ_r as the wavelength *initially observed* immediately following the onset of the instability. If λ_f is sufficiently close to λ_n, the system is spatially locked and one observes a perfectly regular pattern of wavelength $\lambda_r=\lambda_f$.

The initial response characteristics of the interface are summarized in the mode diagram of Fig. 3. We have adopted the same format as Ho and Huang [3]: The ratio of response wave number k_r to forcing wave number k_f is plotted as a function of the ratio of forcing wave number k_f to natural wave number k_n. The symbols refer to different inclination angles as indicated. It should be emphasized that this diagram only determines the initial spatial periodicity of the interface immediately beyond the time of onset. When the external wave number is sufficiently close to the natural wave number one obtains a locked configuration at $k_r=k_f$ as previously discussed. Below a critical value of k_f/k_n, the response wave number k_r jumps discontinuously to $2k_f$ and remains fixed at this value for a finite range of forcing wave number k_f. Additional integer step increases in k_r/k_f are observed at lower values of k_f. Thus, the flow response near onset can be summarized by the single relation $k_r=mk_f$, where $m=1,2,3,4$ is an integer that depends on the forcing wave number k_f. The diagonal line $k_r=k_n$

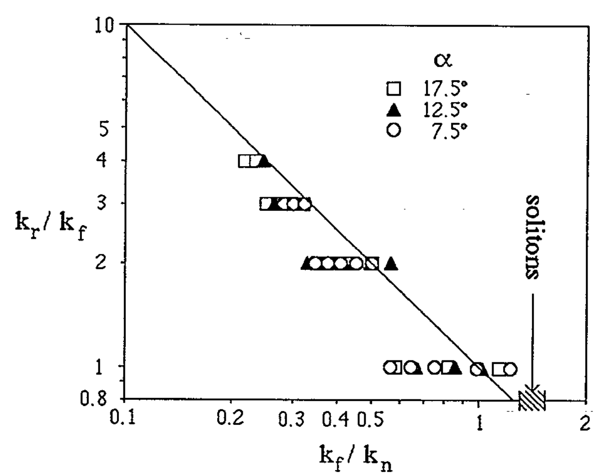

FIG. 3. Mode diagram of accelerating shear layer under forced conditions. See text for definitions of k_n, k_f, k_r. The diagonal line corresponds to $k_r=k_n$. Phase solitons are observed in the hatched region.

has also been drawn in Fig. 3. Note that the interface tends to select an integer multiple of k_f which is sufficiently close to k_n. It is also of interest to notice that the steplike nature of the response wave number curve is observed for all tilting angles, i.e., for all accelerations. However, the values of k_f at which steplike changes in k_r take place do depend on the particular angle α. This is to be expected since the natural wave number k_n decreases with increasing α. In order to take these variations into account the wave number k_f has been scaled with respect to k_n. The mode diagram of Fig. 3 is therefore valid for all angles α. It is the counterpart of the mode diagram in frequency space obtained by Ho and Huang [3] for spatially evolving mixing layers. One only needs to substitute the wave numbers k_r, k_n, and k_f for the frequencies f_r, f_n, and f_f.

The temporal development of the spatial pattern beyond onset is strongly affected, in modes $m=2,3,4$, by the presence of the subharmonic component $k_f=k_r/m$. As shown in Fig. 4, the initially periodic interface of wave number k_r exhibits at a very early stage in its evolution strong amplitude modulations at the forcing wave number k_f: The structures are stronger below the obstacles than at other streamwise locations [Figs. 4(b)–4(d)]. As in spatial mixing layers [3], this amplitude modulation ultimately leads to the formation of structures at the subharmonic wave number k_f. For instance when $m=3$, one observes the temporal evolution displayed in Figs. 5(a)–5(c): Three wave crests ultimately merge to form an arrangement of periodically distributed large structures at the subharmonic wave number $k_f=k_r/3$. When

a)
b)
c)
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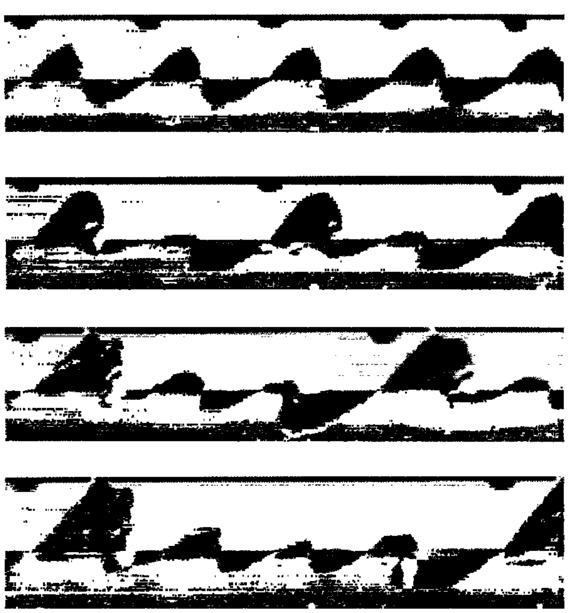

FIG. 4. Interface deformation 0.3 sec after onset of instability. $\alpha=12.5°$, $\lambda_n=4$ cm. (a) $\lambda_f=\lambda_n$. (b) $\lambda_f=2\lambda_n$. (c) $\lambda_f=3\lambda_n$. (d) $\lambda_f=4\lambda_n$.

a)
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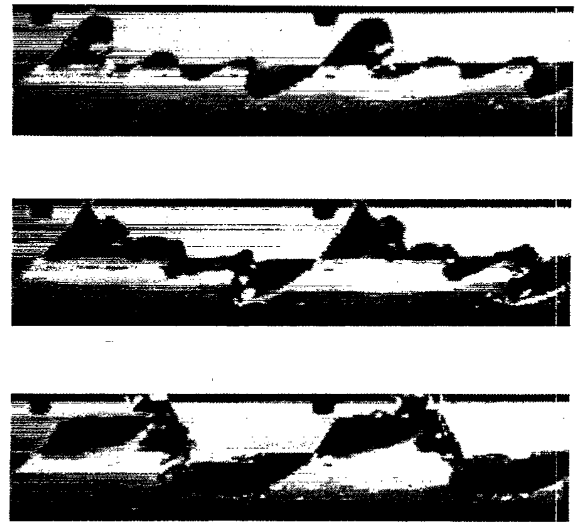

FIG. 5. Temporal evolution of interface. $\alpha=12.5°$, $\lambda_f=3\lambda_n$. (a) $t=0.36$ sec. (b) $t=0.52$ sec. (c) $t=0.76$ sec. The reference time $t=0$ is the onset of instability.

$m=1$, no subharmonic modulations are present and a locked periodic stationary pattern is obtained. The merging of more than four waves ($m>4$) is only observed in strongly accelerating flow for $\alpha>10°$. The forcing wave number k_f is then much lower than k_n, and it is only weakly unstable according to linear theory [7]. High forcing levels are therefore necessary to trigger its growth. In the present configuration, such high levels are achieved only for high accelerations where large velocities are induced above the obstacles. Here again the resulting dynamics is analogous to spatial mixing layers: At low frequencies, sufficiently high forcing levels produce the merging of many individual vortices into a single larger structure, the phenomenon being known as "collective interaction" [3].

There remains to examine the nature of the response when the forcing wave number exceeds the natural wave number, i.e., in the domain $k_f/k_n>1$ of the mode diagram (Fig. 3). If k_f is within the range $1<k_f/k_n<1.2$, the interface deformation is still locked in the first mode at $k_r/k_f=1$. For larger values of k_f, however, the flow response becomes spatially disordered: The competition between the wave number k_f and k_n gives rise to irregularly spaced structures of varying sizes. We have found no evidence of locked modes in the vicinity of $k_f/k_n=2$. In this particular accelerating flow configuration, it appears that locked regimes are only possible when $k_f/k_n<1.2$. Nonetheless interesting patterns have been detected in the small transition region of forcing wave numbers (hatched region of Fig. 3) separating the locked range $k_r/k_f=1$ from the disordered regime. Within this domain of parameter space, the flow response maintains a certain degree of order: It is characterized by large *re-*

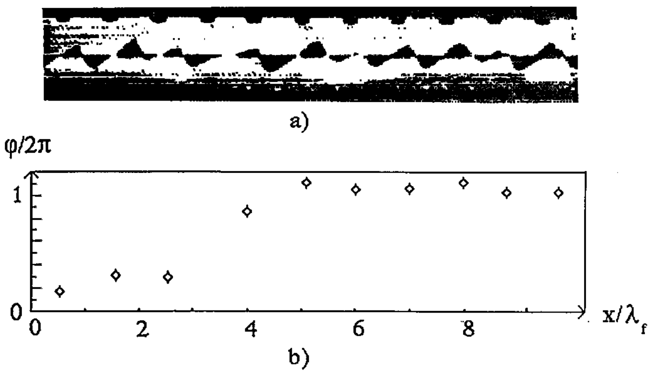

FIG. 6. Phase soliton, $\alpha=12.5°$, $k_f/k_n=1.48$. (a) Interface deformation. (b) Phase $\phi/2\pi$ as a function of streamwise distance x.

gions of the commensurate state $k_r/k_f=1$ that do not extend over the entire apparatus but are separated by zones of local expansion of the basic pattern. An example is given in Fig. 6(a): The interface follows the imposed spatial periodicity in the right and left areas of the image but a larger structure straddles two obstacles in the vicinity of $x/\lambda_f=4$. The nature of this local imperfection can be ascertained by defining the phase function $\phi_n=2\pi(x_n-n\lambda_f)/\lambda_f$ that specifies the location x_n of the nth wave crest with respect to the position $n\lambda_f$ of the nth obstacle. The variations of ϕ along the stream [Fig. 6(b)] reveal that the imperfection is associated with the existence of a 2π phase jump in the streamwise direction. As k_f is increased beyond $1.2k_n$, stationary solitonlike phase variations appear in a manner that is strongly reminiscent of the experimental observations in a convecting nematic fluid [1]. The existence of phase solitons has been shown to be a generic feature of commensurate-incommensurate transitions in variational continuous dissipative systems [2,8]. The theoretical formulation is based on a phase dynamics description of the evolution of the pattern: As the misfit between the forcing and natural wave numbers is increased, a definite threshold exists beyond which the minimum of the potential energy no longer occurs at the commensurate state but at an incommensurate state composed of phase solitons. These theoretical results pertain to dissipative systems and are not applicable in the present context. We have argued that, in the tilting tank experiments, viscous dissipation is negligible over the time scales involved and the observed dynamics are essentially conservative. Yet no oscillatory regimes have been observed, as is usually the case in conservative systems. This is probably due to the fact that energy is being continuously injected into the disturbance field through the accelerating shear, thereby masking the conservative nature of the dynamics. The role played by acceleration has not been entirely elucidated and work is in progress to analyze the constant shear case in which the tank is brought back to its initial position.

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