Absolute and convective nature of the Eckhaus and zigzag instability with throughflow

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The nature of the Eckhaus and of the zigzag instability is investigated for a periodic basic "flow" (a y-periodic Stokes solution) in the presence of a transverse or a longitudinal mean flow using the two-dimensional extension of the absolute instability criterion. For each flow orientation, stability diagrams are obtained numerically and analytically for a simple amplitude-equation model considering both the Eckhaus and the zigzag instability. Analytical results extend and correct a previous analysis by Müller and Tveitereid.[1] In particular, for a longitudinal flow, the Eckhaus instability is convective near its instability threshold and the absolute destabilization occurs at a finite wave number. Similar results hold for the zigzag instability for a transverse throughflow which is convective near threshold. In the presence of an arbitrarily oriented mean flow, the absolute threshold for the Eckhaus instability is also numerically determined. Implications of these results for real experiments are discussed. [S1070-6631(99)00511-5]

I. INTRODUCTION

When considering the primary instability of an open flow, such as a wake, jet, mixing layer, or boundary layer over a flat wall or concave wall (Tollmien–Schlichting wave or Görtler instability), it is well known that one has to refer to the concept of absolute and convective instability.[2–4] i.e., consider not only the growth of initial perturbations but also their ability to withstand the throughflow. If the impulse response decays to zero at a large time at any fixed location in the laboratory frame, while growing exponentially in some uniformly moving frame, the flow is said to be convectively unstable, whereas it is said to be absolutely unstable when the impulse response grows exponentially at any fixed location in the laboratory frame. In this case the flow is likely to exhibit a self-sustained oscillation due to the saturation of the primary absolute instability as seems to be the case in wakes,[5–8] hot jets[9,10] and mixing layers with counter flow.[11] This behavior contrasts with the convective case where perturbations continuously fed in at the inlet of the unstable flow are amplified throughout their downstream journey (homogeneous jet, coflow mixing layer). The same phenomena are active in closed flows if traveling waves are destabilized, as in binary convection.[12–14]

Self-resonant flows (absolutely unstable flows) or convectively unstable flows subject to a regular forcing usually give rise to a saturated state that consists of a periodic structure either in the direction of the flow (von Kármán-street, single row of vortices in mixing layers) or transverse to it (low-speed streaks, Görtler vortices) or with an arbitrary orientation (inclined shedding behind a bluff body). Naturally. one has to consider the stability of this periodic flow and, once more, the dynamics of this secondary instability will depend on its absolute/convective nature. This question, which is more difficult than for the primary instability since the basic flow is periodic in space, has been recently answered using Floquet theory by Brevdo and Bridges.[15] A second source of difficulty comes from the arbitrary orientation of the basic flow with respect to the pattern, which makes it necessary to consider the propagation of the secondary instability in two dimensions.[15–17]

Close to the threshold the dynamics of a flow for which the primary instability breaks translational invariance may be described by the Ginzburg–Landau equation. The saturated periodic primary structure is described by a Stokes solution that may be subject to the Eckhaus or the zigzag secondary instability. For the Ginzburg–Landau model, a change of variables transforms the Floquet problem for the stability of the Stokes solution into a standard problem with constant coefficients. Huerre[18] has determined the nature of the Eckhaus instability for small amplitude Stokes solutions when a transverse flow is added. In Ref. 1, Müller and Tveitereid have restricted their study to transverse flow for the Eckhaus instability and to longitudinal flow for the zigzag instability. The present study extends this pioneering work to arbitrary orientation of the flow. For the first time, complete stability diagrams are presented for transverse or longitudinal throughflow, for both Eckhaus and zigzag instability.

Following Müller and Tveitereid,[1] we shall consider the Newell–Whitehead equation,[19,20] which describes the formation of an *arbitrarily oriented* periodic state issuing from the primary instability of an extended system. For example let us consider Rayleigh–Bénard convection with an externally imposed mean flow.[21] When a free-slip condition is assumed, the equation for the amplitude A of convection rolls nearly aligned along x with a wave number along y close to K_c

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(where K_c is the wave number of maximum growth rate). reads

$$(\partial_t+u\partial_x+v\partial_y)A=[\mu+(\partial_y-i\partial_x^2/2K_c)^2-|A|^2]A. \quad (1)$$

The amplitude equation has been rigorously derived (from the Boussinesq approximation) only close to the threshold and without external shear flow added to the Rayleigh–Bénard problem, by Zippelius and Siggia.[22] It is valid for large Prandtl number, since a term coupling with the vertical vorticity of the mean flow should be be added in Eq. (1) for small Prandtl numbers. However, this term affects only the phase and we will not take it into account, since the main physical effect we wish to highlight is already present in Eq. (1). The external shear flow driven by a pressure gradient or moving boundaries appears in the convective nonlinearity $(\mathbf{u}\cdot\nabla)$ and does not affect the basic state since it depends only on the vertical coordinate, but it does affect the fluctuations, since the term $u\partial_x+v\partial_y$ appears in their evolution equation. Here we have made the simplification of applying free-slip conditions on the upper and lower plates, as considered at first in the studies of mean-flow effects in Rayleigh–Bénard convection;[22] therefore, the throughflow of uniform velocity (u,v) just corresponds to a Galilean transformation of the standard Newell–Whitehead equation; μ is the departure from the threshold. As discussed by Müller and Tveitereid,[1] if no-slip upper and lower boundary conditions are considered, then all the coefficients in Eq. (1) should be assumed complex and extra terms breaking the rotational invariance in the advected frame should be added to Eq. (1). It should be stressed that Eq. (1) is not fully derived from a systematic expansion in any small parameter. Certainly other nonlinearities may be added to the same order. We have merely tried to account qualitatively for the effects of mean advection in convection (see Ref. 1 for a discussion).

In the present form, Eq. (1) may also receive an alternative interpretation since in the absence of mean advection, it describes rigorously the asymptotic evolution of the Green's function (the impulse response) on a particular ray $\mathbf{x}/t=(u,v)$, with t, the time from the initial impulse applied at $\mathbf{x}=0$. Determination of the selected frequency and wave number on each ray would then enable us to reconstruct the entire wave packet.[15–17]

Equation (1) admits nonlinear Stokes solutions $A=A_k\exp(ik(y-vt))$ representing convection rolls with a wave vector $K=(0,K_c+k)$ and a saturated amplitude $A_k=\sqrt{\mu-k^2}$. We perturb this solution by

$$\delta A=e^{ik(y-vt)}[\delta A_1 e^{i\mathbf{q}\cdot\mathbf{x}-i\omega t}+\delta A_2 e^{-i\mathbf{q}^*\cdot\mathbf{x}+i\omega^* t}]+\text{c.c.},$$

where the star denotes complex conjugation. We obtain a system of two equations in δA_1 and δA_2 which leads to the dispersion relation $D(\omega,q_x,q_y)$ equivalent to that given by Müller and Tveitereid[1]

$$D(\omega,q_x,q_y)=i\omega-i(uq_x+vq_y)-A_k^2-(U_++U_-)/2$$
$$+(A_k^4+(U_+-U_-)^2/4)^{1/2}=0. \quad (2)$$

with $U_\pm=(k\pm q_y+q_x^2/2K_c)^2-k^2$.

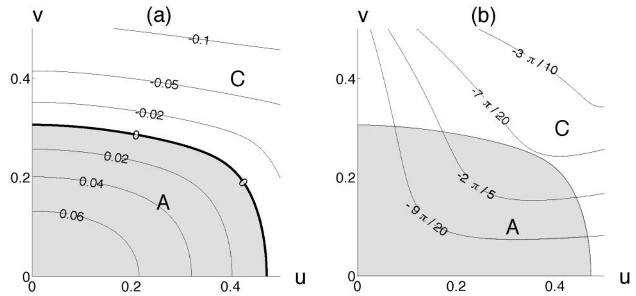

FIG. 1. Isolines of the absolute growth rate ω_i^0 (a) and angle of the wave vector q_0 with the x axis (b) as a function of the velocity (u,v).

II. ECKHAUS INSTABILITY

We will first consider the Eckhaus instability and, therefore, restrict ourself to $k>0$ to avoid any interaction with the zigzag instability. The classical Eckhaus instability in one dimension $(q_x\equiv 0)$ occurs at $k=1/\sqrt{3}$ when $\mu=1$ for $q_y=0$. In two dimensions, we see from Eq. (2) that the instability occurs at $k=1/\sqrt{3}$ when $\mu=1$, simultaneously on the parabolas $q_y=\pm q_x^2/2K_c$.

Let us now determine for a fixed μ and k in the Eckhaus unstable domain, the limiting value in the advection-velocity (u,v) plane for which the Eckhaus instability is absolute. According to the theoretical proof given by Brevdo,[23] one has to look for double saddle points $(\omega^0,q_x^0,q_y^0)\in\mathbb{C}^3$ verifying the three complex relations (plus a pinching condition not made explicit here but similar to the one-dimensional case[2,3])

$$D(\omega^0,q_x^0,q_y^0)=0, \quad (3a)$$

$$\partial D/\partial q_x(\omega^0,q_x^0,q_y^0)=0, \quad (3b)$$

$$\partial D/\partial q_y(\omega^0,q_x^0,q_y^0)=0. \quad (3c)$$

The absolute growth rate is then defined as $\omega_i^0=\Im(\omega^0)$ and the flow will be absolute when $\omega_i^0>0$. In the present problem ω^0 is a function of (u,v,μ,k,K_c), rescaling of time and space allows us to remove two of the parameters while keeping the diffusion coefficient as unity. Therefore, K_c and μ are set to one to draw Fig. 1, which may be rescaled for any other value of μ. In Figs. 2 and 3, μ has been kept to facilitate comparison with experiments. System (3) with the pinching condition is solved numerically using Matlab. Results are shown in Fig. 1 for $\mu=1$, $k=1/\sqrt{3}+1/10$.

These parameter values are in the Eckhaus unstable region. The value of ω_i^0 as a function of (u,v) is given in Fig. 1(a): The heavy line represents the zero iso contour that delineates the absolutely unstable region (because of the symmetry only one quarter of the figure has been reproduced). Figure 1(b) represents the angle of the wave vector with the horizontal (i.e., $\theta=\text{Arctg}(\Re(q_y^0)/\Re(q_x^0))$) versus (u,v).

For an arbitrary orientation of the mean flow with respect to the pattern, the transition from convective to absolute instability given in Fig. 1(a) occurs for inclined wave making an angle with the pattern as large as $\sim 7\pi/20$ [maximum of $\theta(u,v)$ on the curve $\omega_i^0(u,v)=0$]. It is striking to notice that, when the advection velocity is directed along the

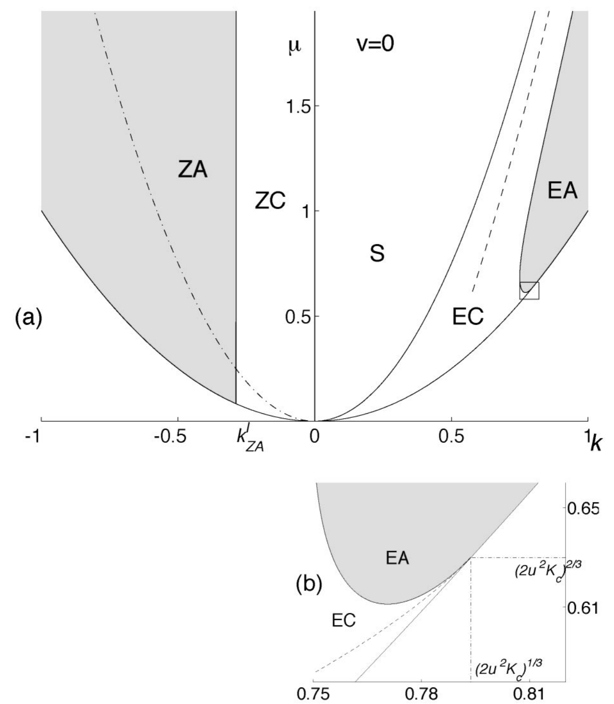

FIG. 2. Stability diagram for a longitudinal flow ($v=0$), with $u=0.5$ and $K_c=1$. The pattern exists for $\mu>k^2$. For the Eckhaus instability, it is stable (S) when $\mu>3k^2$, convectively unstable in the (EC) region and absolutely unstable in the (EA) shaded region. The dashed line on the right side represents the asymptotic threshold of absolute instability (5) for large μ. The enlargement (b) presents the asymptotic threshold (4) (dashed line) for which the Eckhaus instability of the rolls becomes absolute near the neutral curve $\mu=k^2$. For the zigzag instability, the pattern is convectively unstable (ZC) for $k^l_{ZA}<k<0$ and absolutely unstable (ZA shaded region) for $-\sqrt{\mu}<k<k^l_{ZA}$.

y axis or on the x axis, the angle of the absolute wave number is $\pi/2$ and therefore corresponds to a pure compression wave. This is natural when the advection is normal to the rolls but this is surprising when the advection is parallel to the rolls. For this absolute versus convective instability with a longitudinal flow ($u\neq 0$, $v=0$) the solution of the paradox lies in the imaginary part of q_x^0, which stays finite whereas the real part goes to zero when v goes to zero. It is important to keep in mind, conversely to the assumption made in Ref. 1, that $q_x\neq 0$ for the Eckhaus instability of a longitudinal pattern, the true solution being an uniform compression wave in y ($q_y^0\in\mathbb{R}$) growing exponentially in x ($q_x^0\in i\mathbb{R}$).

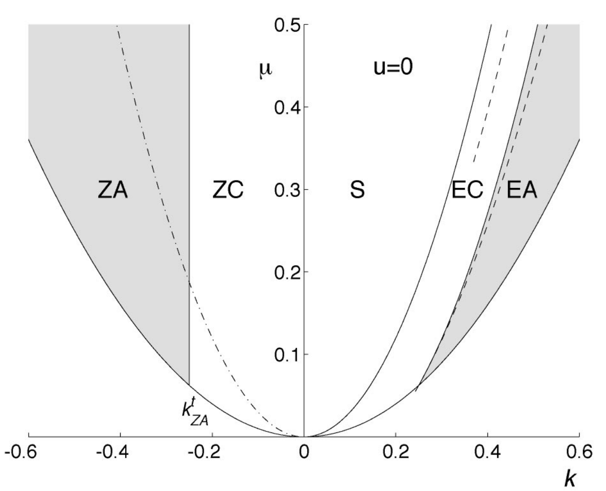

FIG. 3. Same as Fig. 2 but for transverse flow ($u=0$) i.e., with pattern wave vector K_c+k parallel to the throughflow, with $v=0.5$ and $K_c=1$. The dashed lines represent the asymptotic boundaries between EC and EA regions for large μ [Eq. (7)] and for small A_k [Eq. (6)].

A. Longitudinal flow

Figure 2 presents the complete stability diagram of the pattern with wave vector K_c+k in the y direction with a perpendicular throughflow ($u=0.5$, $v=0$). The results concerning the Eckhaus instability are obtained for $k>0$ whereas the results for the zigzag instability ($k<0$) will be detailed in the next section. For $k>0$, the boundary between the convectively (EC) and absolutely (EA) unstable regions is computed numerically by solving system (3). This boundary has also been determined analytically in two limits.

(1) Near the neutral curve of the primary instability $\mu=k^2$: systems (3b) and (3c) may be written as a polynomial equation of degree 6 in q_x^0 and solved[24] using the fact that the amplitude A_k of the pattern is small: The condition $\omega_i^0(q_x^0,q_y^0)=0$ expanded at first order in A_k determines the boundary between absolute and convective regions when $\mu\to k^2$

$$\mu\sim k^2+\frac{2^{5/2}}{3^{3/2}}\frac{(u\sqrt{2K_c}-k^{3/2})^{3/2}k^{5/4}}{u\sqrt{K_c}}, \tag{4}$$

which is represented by a dashed line in the close-up Fig. 2, and is confirmed with numerical computation.

(2) For large enough control parameter μ *and order-one velocity* (equivalent to $\mu=1$ and small enough u), the solution of system (4) can be extracted using the scaling issuing from a small-u expansion,[25] and the boundary between absolute and convective Eckhaus instability of the pattern up to second order in u reads

$$\mu\sim 3k^2-(1+\sqrt{37})u^2/8k. \tag{5}$$

Therefore, the classical boundary $\mu=3k^2$ is affected by the throughflow and shifted by a quantity proportional to u^2/k at large k. This result is in agreement with the idea that the Eckhaus instability propagates from a localized perturbation at a finite speed in the transverse direction.

B. Transverse flow

This behavior is radically different for the Eckhaus instability of a transverse flow ($u=0$, $v\neq 0$) (Fig. 3 for $k>0$) for which $q_x=0$ (both real and imaginary parts zero) and q_y is complex with nonzero real and imaginary parts. The boundary of absolute (EA) Eckhaus instability is determined by solving system (3) numerically. The dashed line in Fig. 3 given by

$$\mu=2k^2-v^2/4, \tag{6}$$

represents the asymptotic boundary between absolute and convective regions near the neutral curve $\mu=k^2$, as found by Huerre[18] by a small-A_k expansion similar to (4). A large-μ-expansion with order-one v, similar to (5) (equivalent to a small-v expansion) allows determination of the shift from the classical Eckhaus boundary $\mu=3k^2$, which is proportional to $k^{4/3}v^{2/3}$ as found in Ref. 1

$$\mu=3k^2-\alpha k^{4/3}v^{2/3}, \quad (7)$$

where $\alpha=(7\sqrt{7}-17)^{1/3}/2^{4/3}$.

III. ZIGZAG INSTABILITY

For completeness, let us now determine the stability diagrams of the longitudinal ($v=0$) and of the transverse flow ($u=0$) subject to the zigzag instability, when $k<0$. The domain of absolute instability (ZA) of the zigzag unstable ($k<0$) pattern has been determined analytically by solving (3). It should be noted that for $k<-\sqrt{\mu/3}$ the two-dimensional instability modes are of a hybrid type between Eckhaus and zigzag. The present analysis takes into account the entire dispersion relation and, therefore, does not filter out any instability mechanism. For simplicity and since the crossing of the Eckhaus domain frontier $k<-\sqrt{\mu/3}$ does not modify the absolute instability threshold, the instability for $k<0$ will be simply refered as the zigzag instability.

A. Longitudinal flow

For a longitudinal flow ($u\neq 0$, $v=0$), the solution of system (3) is $q_y^0=0$ and $q_x^0=(\delta^{(1/2)}-uK_c^2/2)^{(1/3)}e^{i\pi/6}+\delta^{(1/2)}+uK_c^2/2)^{(1/3)}e^{-i\pi/6}$, with $\delta\equiv u^2K_c^2/4-8k^3K_c^3/27$ (there is a symmetric solution found by replacing $\pi/6$ by $5\pi/6$). The boundary for which the zigzag instability becomes absolute is given by the condition $\omega_i^0=0$

$$k_{ZA}^l=-\alpha u^{2/3}K_c^{4/3}, \quad (8)$$

where α is the same constant as in (7). This limit, shown in Fig. 2, is the one found by Müller and Tvetereid.[1] Together with the new results for the Eckhaus instability described in the previous section, it gives the complete stability diagram with a longitudinal flow (Fig. 2).

B. Transverse flow

As for the Eckhaus instability with a longitudinal flow, the claim in Ref. 1 that, no matter what the velocity v the zigzag instability with a transverse flow is absolute, is not in agreement with our result since we have performed a two-dimensional analysis from which we obtain a shift of the absolute zigzag threshold given by Müller and Tveitered's one-dimensional analysis. The absolute instability (ZA) region (in Fig. 3) is bounded by the vertical line

$$k_{ZA}^l=-v/2, \quad (9)$$

given by the pinching-saddle point solution with q_x^0 real and q_y^0 pure imaginary ($q_x^0=\pm\sqrt{-2kK_c}, q_y^0=-iv/2$). When v goes to zero, the boundary for the absolute instability of the pattern goes to the classical boundary $k=0$ for the linear instability of the rolls.

These solutions and the one obtained in the previous section yield the complete absolutely unstable domain (Fig. 3) of a pattern with a transverse flow.

IV. EXTENSION TO OPEN FLOWS

All the above derived results have been obtained for a complex amplitude equation with real coefficients; it is not believed that the extension to complex coefficients will modify radically the phenomenology, as already demonstrated by us in an exhaustive study of the secondary instability of saturated waves governed by the one-dimensional complex Ginzburg–Landau equation.[26]

Although derived from a model strictly valid for Rayleigh–Bénard convection with a small throughflow, the present analysis enables us to propose experimental validations and interpretations of observations.

These new results for the Eckhaus unstable pattern with a longitudinal throughflow and for the zigzag unstable pattern with a transverse flow show that a throughflow always makes it necessary to discriminate between absolute and convective instability of the pattern. In particular, when Rayleigh–Bénard convection rolls are formed parallel to the inlet boundary (transverse throughflow), the zigzag instability should affect the flow when the wave number is smaller than the absolute critical value given by Eq. (9). The zigzag instability is thus postponed to the region where it is absolute and at the threshold, the absolute wavelength that dominates the flow is finite. This should be easily verified in Rayleigh–Bénard experiment with throughflow by forcing at the inlet boundary the appearance of rolls with a period larger than the natural one (for example by modulating in time the inlet temperature). Above a low forcing frequency threshold, a zigzag instability, periodic along the rolls and growing in the mean flow direction (transverse to the rolls), should become self-sustained, the spatial period being finite at threshold. Conversely, if longitudinal Rayleigh–Bénard rolls (normal to the inlet boundary) are forced at a cross-stream wavelength controlled by the inlet boundary condition (modulation of the temperature along the span), they should be subject to an Eckhaus instability at finite perturbation wave number when the basic flow periodicity is small enough.

Extension to wakes, jets or boundary layers is more hazardous since the model does not apply in these cases, and since temporal instability is known to involve other mechanisms such as pairing and translative instabilities[27] that occur at a finite wave number and that are only loosely connected to Eckhaus and zigzag instabilities. However, Secondary instability of wakes such as the so-called mode *A*, that is known from numerical analysis to be self sustained,[28] may still be a remnant of the zigzag instability that becomes absolute. Boundary layer flows may also show remnants of these phenomena, and secondary instability in Görtler flow (flow over a concave wall) may show the existence of a self-sustained Eckhaus instability with a finite wave number that will induce periodic oscillations and transverse wandering of the longitudinal vortices. Controlling the wave number of the Görtler vortices by periodic cross-jets or wall roughness elements should show both the absolute Eckhaus instability, when the vortices are too close together, and the absolute zigzag instability when the vortices are too far apart. As far as turbulence control is concerned, an optimal longitudinal vortex spacing will lie in between these two absolute secondary instability limits. Of course the latter reflections are only speculative since open shear flows are not rotationally invariant and the phenomenological amplitude equation to consider differs from Eq. (1). Furthermore the use of amplitude equation for open flows might not be appropriate since shear flows are subject to finite wave number secondary instabilities that depart strongly from the phase instabilities (Eckhaus and zigzag instabilities), such as pairing, translative, hyperbolic or elliptic instabilities.[29,30] A direct analysis of the convective or absolute nature of these secondary instabilities should be conducted for each particular primary flow. A numerical procedure that makes such a goal feasible, based on a direct computation of the impulse wave packet, has recently been proposed by Brancher and Chomaz[31] and used to determine the absolute instability threshold for the pairing and the translative instabilities of a infinite row of vortices described by the Stuart model.[32]

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