# Vertical diffusion of the far wake of a sphere moving in a stratified fluid

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The far wake of a sphere towed horizontally in a linearly stratified fluid with Froude numbers $F \in (0.25,12.7)$ and Reynolds numbers $\mathrm{Re} \in [150,30\,000]$ is investigated. Regardless of the initial Froude number, the wake becomes quasi two dimensional at times large compared with the Brunt–Väisälä period. For $F \lessdot 4.5$, the horizontal motions are coherent over the whole depth of the wake. The wake takes the form of a regular von Kármán street for $F \lessdot 1.5$ but is irregular at larger $F$. For $F \gtrdot 4.5$ the far wake consists of, vertically incoherent, horizontal motions in several layers. The transition occurs smoothly by a vertical decorrelation of the horizontal motion in the depth of the far which starts at $F=4.5$. The most novel result is that the horizontal velocity and vertical vorticity diffuse vertically in a time much shorter than predicted by a viscous diffusion law. The ratio of the observed diffusion time to the viscous diffusion time is expected to depend on Reynolds number as is indicated by a simple model.

## I. INTRODUCTION

Stratified flows are known to possess an intricate dynamics with turbulent motions interacting with gravity waves. For Froude numbers, $F=U/LN$, of order one, wave motions and turbulence are strongly coupled because in this case the advection frequency $U/L$ and the Brunt–Väisälä frequency are of the same order (here $U$ is a typical velocity, $L$ a length scale, and $N$ the Brunt–Väisälä frequency, $N=\sqrt{-g/\rho\ d\rho/dz}$, with $\rho$ the density, $z$ the vertical coordinate, $g$ the gravity module). However, when the Froude number is small, it can be used as a small parameter to perform asymptotic expansions in order to separate wave motions from turbulent motions. This condition is met for instance in the late stages of stratified turbulence decay of which the far wake is a good example. Riley *et al.*[1] and Lilly[2] showed that when $F$ is small, waves generate motions on a rapid time scale $N^{-1}$ with an order of 1 vertical motion, whereas turbulent motions occur over a slow time scale $N^{-1}/F$ with an order $F^2$ vertical velocity component. These turbulent motions are governed in each horizontal layer by the two-dimensional Navier–Stokes equations. This decoupling of motion has been observed experimentally in the wake of bodies of revolution (Lin and Pao,[3] Hopfinger[4]) and has also been demonstrated numerically for homogeneous stratified turbulence (Métais and Herring[5]). In the later case, the late stages of turbulence decay are characterized by the so-called pancake turbulence with quasi-two-dimensional motions in several uncorrelated layers. This formation of two-dimensional motions in layers is also familiar to oceanographers and contributes to the deep ocean microstructure. It is also believed to explain observed atmospheric spectra with a $k^{-5/3}$ slope, characteristic of up-scale energy transfer typical of two-dimensional turbulence.[2]

The near wake velocity and wave fields generated by a horizontally moving sphere have been previously reported (Bonneton *et al.*,[6,7] Chomaz *et al.*,[8–11] Hopfinger *et al.*,[12] Lin *et al.*[13,14]). The present paper is concerned with the structure of the far wake and with the vertical diffusion of the quasi-two-dimensional motions of the far wake. By means of flow visualizations of a single wake, exploring a wide Reynolds–Froude number parameter space, it has been possible to determine that the far wake can be either regular and single layered, irregular, and single layered, or irregular and multilayered, depending mainly on the Froude number of the flow. The single and multilayered far wake structures are well demonstrated by visualizations of the three interacting wakes.

## II. EXPERIMENTAL APPARATUS

Confinement in the vertical, as well as, in the horizontal direction may affect the far wake evolution. In order to keep this effect small we used a large towing tank which measures 22 m long, 1 m deep, and 3 m wide. A small tank, $0.5\times0.5\times4$m, was used in the study of the small Froude number wakes where confinement effects are of lesser importance because the Froude number, based on the depth of the tank is smaller. The spheres had radii $R$ of 1.12, 2.5, or 3.6 cm (a "Pétanque" bowl). The towing speed was varied between 5 and 40 cm/sec in the larger tank and between 1 and 40 cm/sec in the smaller tank, giving a maximum Reynolds number of about 30 000. The stratification was realized by a computerized process, mixing variable quantities of clear water with a saturated salt solution at the same temperature.[8] The process takes eight hours to fill the large tank and approximately five tons of salt are needed each time (part of it is recycled). This linear stratification lasts months because a locally perturbed linearly stratified fluid returns after a few days to the initial stratification and because the diffusivity Di of salt is too small to diffuse the density gradient over one meter during this time ($Di=\nu/800$, where $\nu$ is the kinematic viscosity of water). The surface flux was reduced by adding and maintaining a thin layer (2 cm thick) of fresh water at the free surface.

The Froude number and the Reynolds number in the experiments are known to an accuracy of 2% to 3%. For consistency and identification of the experiments we give the values of $F$ and Re as calculated with the measured parameters (no roundoff). The structure of the wake was visualized by dye which was either deposited on the sphere surface before immersion of the sphere or emitted through small holes in the sphere. Quantitative measurements of the motions in the far wake have been made by marking the fluid before the experiment was started. Either a set of horizontal lines, made of fluorescent dye, was introduced, normal to the motion of the sphere, in a single plane a given distance below the sphere center or a set of vertical fluorescent dye lines regularly spaced was used. When these vertical lines are intersected by a horizontal laser sheet the fluid appears marked by bright spots. The deformation of the horizontal lines gives only access to the longitudinal velocity, whereas, when the vertical velocity is small, the deformation of the vertical lines gives access to the horizontal velocity field. The former technique requires one experiment for each horizontal level, whereas the latter, in one single experiment, allows us to monitor the velocity periodically in different levels, by displacing the horizontal laser sheet vertically and so to scan periodically different levels. Since we wish to know the way in which the horizontal velocity diffuses vertically, it is essential to minimize the sphere support perturbation. We used a three wire configuration[9] in all the measurements. Airfoil-shaped rigid supports have been employed only when it was necessary to inject a large amount of dye at the sphere surface, in order to obtain good illustrative pictures of the wake structure.

## III. EXPERIMENTAL RESULTS

As has been demonstrated in the studies of the near wake and of the lee wave field, the dominant parameter appears to be the Froude number $F=U/RN$ and observations can be classified according to its value and especially according to whether $F$ is larger or smaller than the transition Froude number $F_c \approx 4.5$. The Reynolds number in the range ($1000<\mathrm{Re}<30\,000$) affects only weakly the value of this transition Froude number that separates the flow regime without a spiral mode of the wake from the regime with a spiral mode[11] (low frequency mode, Kim and Durbin,[15] 1988). The structure of the wake is also practically independent of Re in this Reynolds number range. The present experiments have been conducted in the range $0.25<F<12.5$ and $150<\mathrm{Re}<30\,000$. Quantitative results of the vertical diffusion of the far wake have been obtained for $(F,\mathrm{Re})=(3,3500)$ and $(F,\mathrm{Re})=(5,6000)$. The flow regime diagram is presented in Fig. 1 and discussed in the following section.

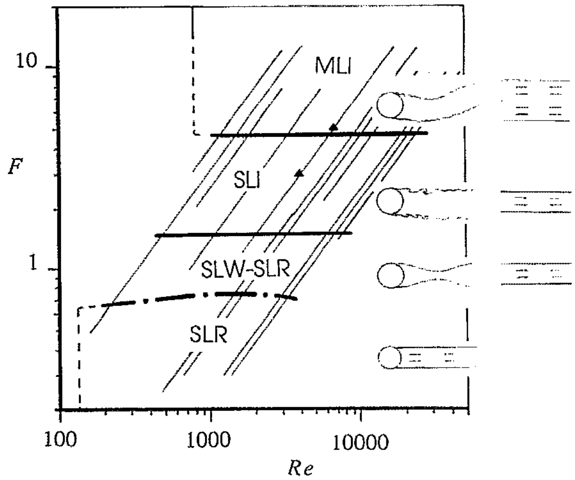


FIG. 1. Flow regime diagram for the far wake of a sphere: multilayer, irregular (MLI) far wake; single-layer, irregular (SLI) far wake; saturated lee wave, single-layer, regular SLW–SLR, far wake; (SLR) far wake. Each thin line indicates a set of about 15 to 20 different couples ($F$, Re).

### A. Flow structure

In Chomaz *et al.*[11] and Lin *et al.*[13] it is shown that, for small Froude number $F<0.7$, a horizontal layer of fluid travels around the sphere and a regular shedding of opposite signed horizontal vortices occurs, similar to a two-dimensional wake. This double row of vortices is shown in the top view picture presented in Fig. 2 where the fluorescent dye, emitted at the sphere surface, is made visible by a horizontal laser sheet passing through the center of the sphere. It stays stable and strongly correlated on the vertical for a long distance downstream. This regime is named SLR since the motion occurs in a single layer and gives rise to a regular von Kármán vortex street (Fig. 1).

In the regime $0.7<F<1.5$, the near wake is controlled by the lee waves. As the lee wave amplitude decreases with $Nt$ (downstream distance), the wake becomes unsteady

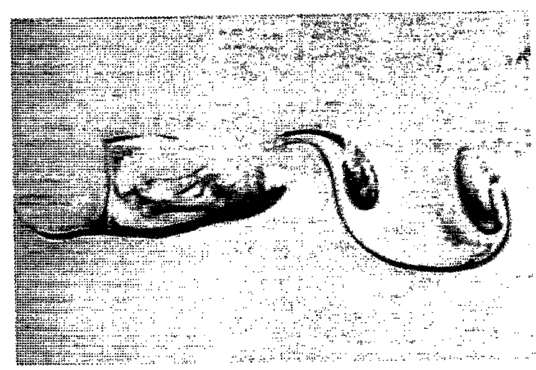

FIG. 2. Top view of the regular von Kármán vortex street formed by shedding of vortices behind the sphere at $F=0.4$, ($\mathrm{Re}=1644$, $R=3.6$ cm). The domain size is $31.2\times 23.2$ cm. The sphere is moving from right to left. The wake is visualized by dye emitted at the sphere surface. The fluorescent dye is excited by a horizontal laser sheet passing through the center of the sphere.

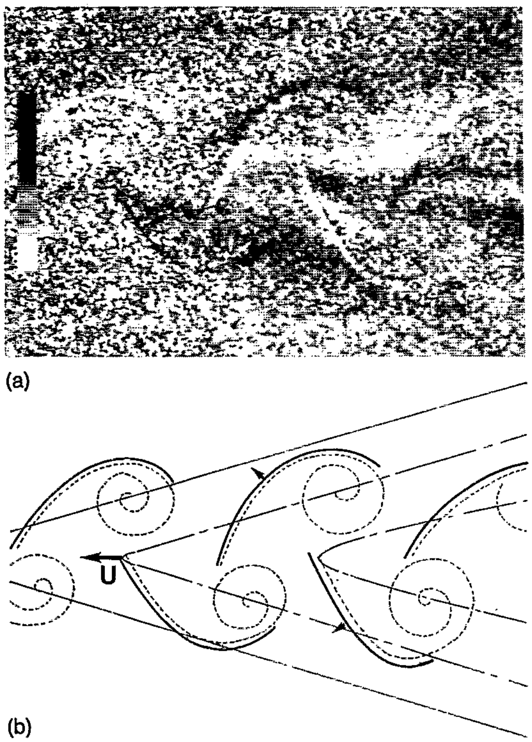


FIG. 3. (a) Shadowgraph top view of the internal wave field at $F=1$ (Re=1948, $R=2.5$ cm), $Nt=33$; (b) sketch of the corresponding wave phase lines: —, lee wave; - - -, waves emitted by the horizontal vortices, and of the corresponding quasi-two-dimensional motions in the wake, ---. The domain size is 24×17 cm. The sphere is moving from right to left.

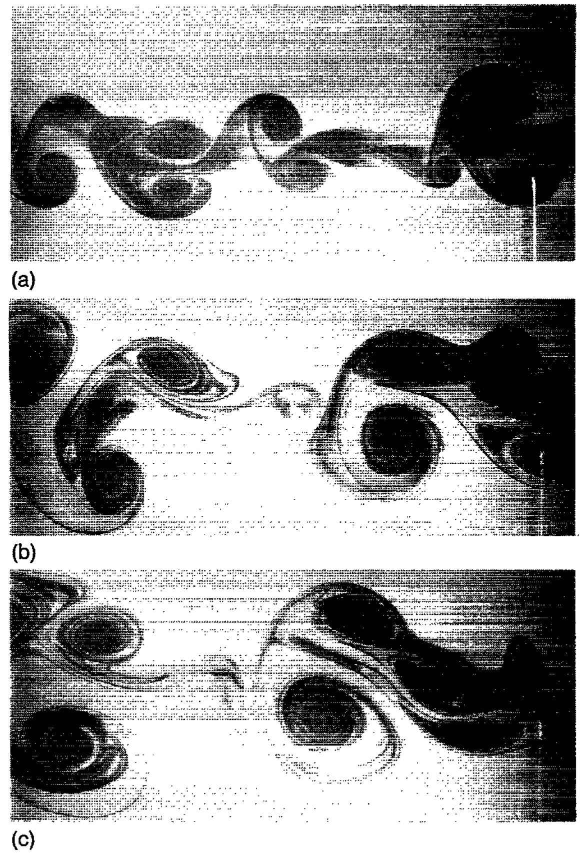


FIG. 4. Time sequence of the evolution of an irregular vortex street formed by the instability of the quasi-two-dimensional wake profile for $F=10/\pi$ (Re=4322, $R=2.5$ cm); (a) $Nt=150$; (b) $Nt=600$; (c) $Nt=1050$. The domain size is 190×127 cm. The sphere is moving from right to left. The image shows a top view of the wake visualized by dye emitted at the sphere surface.

and a regular von Kármán vortex street forms. Shadowgraph top views enable the visualization of the entire internal wave field. It is possible to distinguish in Fig. 3(a), corresponding to $F=1$ and $Nt=33$, the lee wave field with its characteristic hyperbolic phase lines, from internal waves generated by the two-dimensional vortices. These waves are $\lambda$-shaped and coincide with regions of large strain of the two-dimensional motion in the far wake. We have sketched in Fig. 3(b) the wave field and the quasi-two-dimensional motions. This regime is named SLW–SLR since the far wake is similar to the SLR case whereas the close wake is dominated by the saturated lee wave (SLW regime of Ref. 11) with no direct vortex shedding on the sphere (Fig. 1).

In the regime $1.5<F<4.5$, no shedding of quasi-two-dimensional vortices is observed. A double row of irregular alternating vortices forms after the three-dimensional motion is suppressed by the stratification. This regime is named SLI since the motion occurs in a single layer and gives rise to an irregular von Kármán vortex street (Fig. 1). These vortices are preceded by an irregular undulation of the wake. The mechanism of initiation of the vortices is probably a quasi two-dimensional instability of the mean profile with no sharp frequency selection because the forcing by the collapsing three-dimensional turbulence is random. The mean wavelength is near to the most amplified mode. The evolution of such a wake is presented in Fig. 4 for $F=10/\pi$. The resemblance with the numerically computed temporal evolution of a two-dimensional homogeneous wake, initially perturbed by a white noise, is striking (Couder and Basdevant[16]). An interpretation may be that, in the experiment for Re$\geqslant$1000, the decay of the three-dimensional turbulence produces, at the far stage, random forcing similar to the initial noise introduced in the simulation. The far wake spreads horizontally similar to a two-dimensional turbulence front with a spreading rate of order $V$, the characteristic velocity of the eddies of the far wake. The photograph shown in Fig. 4 has been taken in the large tank. Dye was injected at the sphere surface and the wake photographed from above with lighting from the side. It is qualitatively similar to the far wake, obtained in a smaller tank, shown in Hopfinger.[4]

Froude numbers larger than 4.5 define the regime MLI (Fig. 1), where the motion in the far wake is multilayered and irregular. The transition value of $F\approx4.5$ has been identified in relation with a change in the near wake structure.[11] In order to illustrate that the structure of the far wake also depends on whether the Froude number is below or above the transition value of $F\approx4.5$, we visualized the interaction of three wakes of identical spheres towed at the same depth. Figure 5 presents top view photographs with lighting from the side, similar to Fig. 4. In

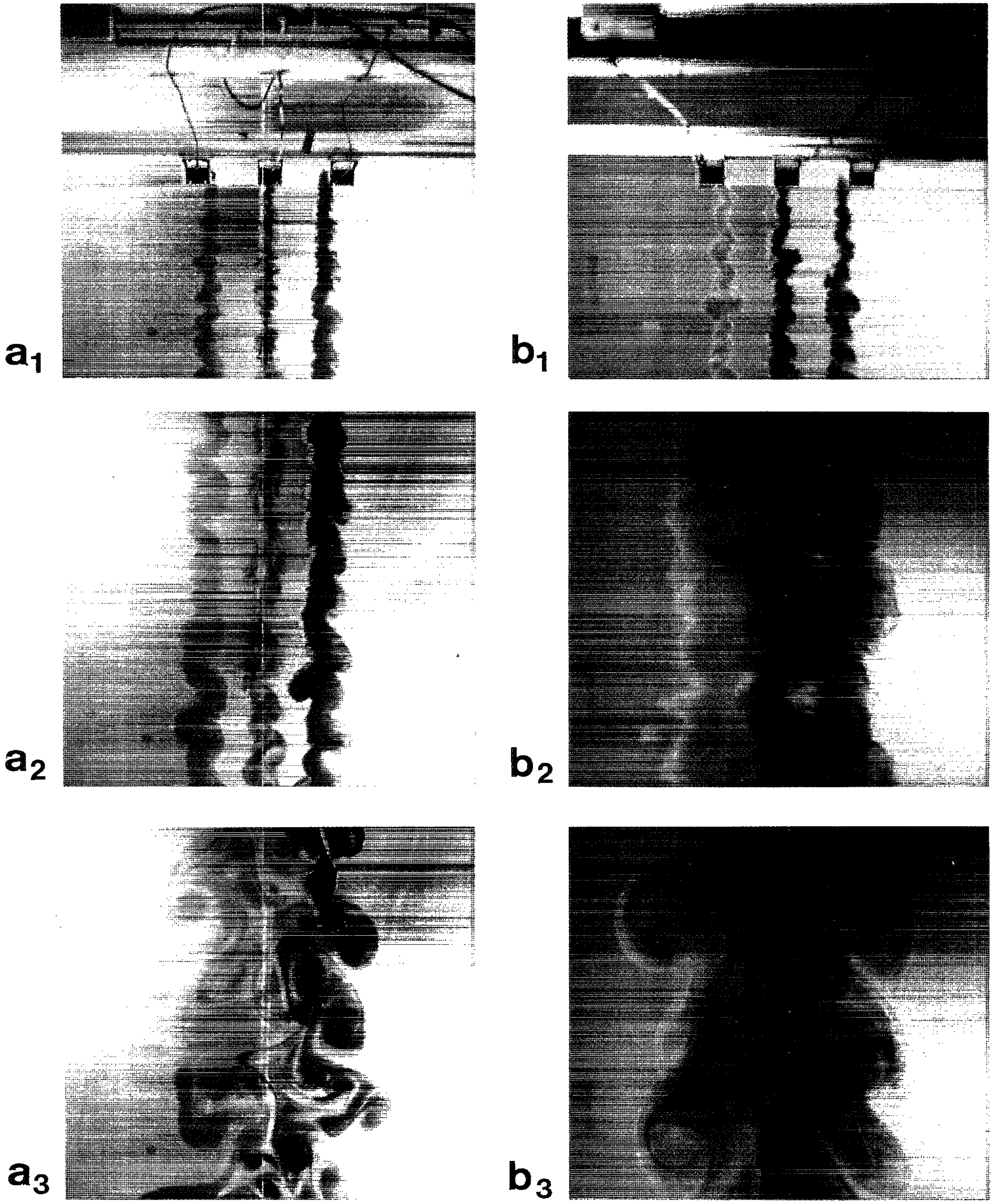


FIG. 5. Interaction of the wake of three spheres ($R=2.5$) at ($a_i$), $F=10/\pi$ (Re=4322) and ($b_i$), $F=40/\pi$ (Re=17288). $a_1$, $b_1$, $Nt=0$; $a_2$, $b_2$, $Nt=37.8$; $a_3$, $b_3$, $Nt=264.6$. In picture (b) several layers have formed in the wake directly from the collapse of the large-scale spiral mode of the three-dimensional wake, resulting in overlapping of the colors due to the optical integration of the picture. The domain size is $216\times200$ cm for the $F=10/\pi$ series and $210\times192$ cm for the $F=40/\pi$ series. The images are top views of the wakes visualized by different dyes emitted at the sphere surface.

Fig. 5(a) the Froude number is $10/\pi$ and in Fig. 5(b) $F=40\pi$. The spheres have a diameter of $R=2.5$ cm and are separated by $6R$. The Froude number was changed by changing $U$ [$U=10$ cm/sec in Fig. 5(a) and 40 cm/sec in 4(b)]. When $F>4.5$, the wake is nearly unaffected by stratification within a downstream distance of at least one spiral mode wavelength $\lambda_1$, having a Strouhal number $\mathrm{St}=2R/\lambda_1\simeq0.2$. This can be seen by writing

$$Nt=x/(0.1\lambda_1F).$$

The wake and more generally, turbulence in stratified fluid, begins to be first affected by stratification when $\mathrm{Nt}\approx2.5$ (Browand *et al.*;[17] Hopfinger *et al.*[12]), giving $x\approx\lambda_1$ when $F=4.5$. When $F<4.5$ and the development of a spiral mode is inhibited by stratification, the turbulent wake, before collapse, has little vertical structure. In this case the far wakes interact in a way similar to two-dimensional turbulence with an essentially horizontal straining and little vertical variation [Fig. 5(a)]. The maximum time of the observation shown in Fig. 5 is $Nt=245$ but the images remain similar for longer times. The interaction of the wakes when $F>4.5$ is quite different. In the case considered in Fig. 5(b) where $F=40/\pi$, the wake can develop two to three whole spiral modes before collapse and has, therefore, much more vertical structure. Consequently, the far wake consists of quasihorizontal, vertically incoherent, motions in several layers. The top view picture, which integrates the whole depth [Fig. 5(b)], shows that the colors are superimposed in the vertical which indicates an apparent mixing.

A possible explanation of the formation of multiple layers may be given by considering the Ozmidov scale $L_0=\sqrt{\epsilon/N}$, where $\epsilon$ is the energy dissipation rate, at the onset of wake structure collapse. The vertical growth of the wake is inhibited when $L_0$ is of the order of the integral scale of the turbulence in the wake. The number of layer would then be related with the ratio of the wake diameter at onset of collapse to the Ozmidov scale. Typically, when $F<4.5$, this ratio is about 2, whereas it starts growing with $F$ when $F>4.5$ and at $F=40/\pi$ it is more like 4 to 5. The value of $F\approx4.5$ is a reference value meaning that at $F<4.5$ the far wake motion is single layered with strong vertical correlation and when $F>4.5$ the vertical correlation decreases and may cease completely, and gradually. For instance, for $F=3$ at $Nt=750$, the vertical correlation coefficient of the vorticity associated with the horizontal motion, measured by the technique explained in the next section, is about 0.8 between two symmetric horizontal planes at the borders of the wake ($z/R=-2.4$ and $z/R=2.4$). In the experiments with $F=5$ at $Nt=750$, the vertical correlation of the motion in the far wake is already much weaker with the correlation coefficient, computed as above, of order 0.6.

### B. Vertical diffusion of the far wake structure

From an early observation,[8] it was apparent that the motion in the far wake diffuses vertically much more rapidly than expected from purely viscous diffusion. For instance after less than 24 h, the motion was coherent on the entire 1 m fluid layer depth. The sphere support is not at the origin of the appearance of vorticity in the entire fluid layer because the Reynolds number based on the wire diameter is less than 40 even at the largest towing speed used (40 cm/sec) and no shedding of vortices occurs from the wires. Besides, all the measurements have been performed in horizontal layers below the sphere where possible perturbations by the support wires are absent.

In order to observe how different horizontal layers are brought into motion, we used the two techniques described in Sec. II. In the first one, the time evolution of a single horizontal layer marked with spanwise oriented horizontal dye lines was monitored during at least 30 min, the time of the whole experiment. The changes in the deformation of the lines indicate the component of fluid motions in the $x$ direction. This technique required one experiment for each $z/R$ position. In the second method, used later on, the fluid was marked by vertical lines of fluorescent dye arranged in a regular square pattern. A layer at position $z/R$ is selected by a horizontal laser sheet making each intersected line appear as a bright spot. Since the movements are slow, a set of 12 layers could be explored sequentially by displacing the laser sheet vertically. Two successive images, taken three minutes apart, allow to determine the motion in each layer from the displacement of the bright spots. In this way the time evolution over 30 min of the movement in 12 different horizontal layers can be explored in a single experiment. More than 300 images were processed with the help of a computer for each experiment. This explains the relatively limited number of experimental runs. To minimize residual motion of the fluid, the experiments have been performed every other day. The remaining residual motion in the tank was measured before the experiment was started and was subtracted from the measured flow field. The residual motion is a low-amplitude large-scale motion (the associated vortices are of the width of the tank, i.e., 3 m wide) and can be subtracted from the wake motions because it does not strongly interact with it.

The motion in three different horizontal planes has been first observed using the horizontal dye line technique for $F=3$ and $\mathrm{Re}\simeq3920$ with the planes of observation distant from the sphere center by $z$ with $z/R=1.2$; 1.8; 3. The precision on each line contour turned out to be rather poor. Nevertheless, a clear change in the line contour has been observed when the residual large scale motion is replaced by stronger, smaller scale motions due to the wake diffusion. For $z/R=3$, this change took place about 6 min after the sphere passage.

Using the vertical dye line technique, the vertical diffusion of the far wake and the evolution of the motion has been measured at 12 positions along the vertical, for $F=3$ and $\mathrm{Re}\simeq3920$ and for $F=5$ and $\mathrm{Re}\simeq6530$. The set of layers scanned by the laser sheet were $z/R=-2.4$; $-1.6$; $-0.8$; 0; 0.8; 1.6; 2.4; 3.2; 4; 4.8; 5.6; 6.4. For $F=3$, in the layers close to the center of the wake $|z/R|<1.6$, the motion exists right after the passage of the sphere and in the layer $z/R=6.4$ away from the wake center the motion stays merely zero up to the end of the experiment (up to

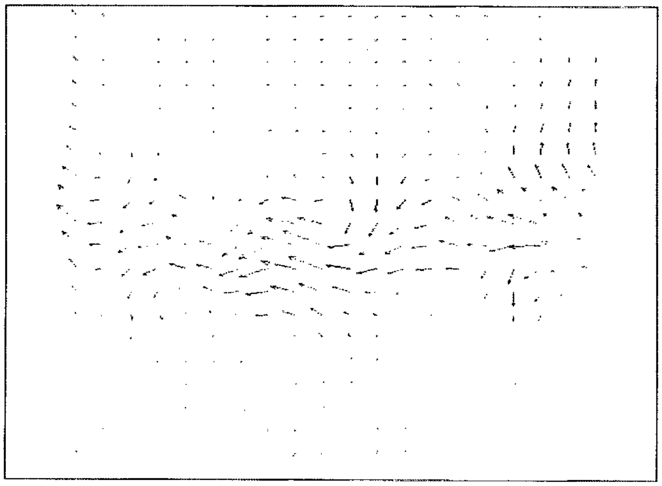

FIG. 6. Velocity field in the far wake at $F=3$ (Re=3920, $R=2.5$ cm) for $Nt=750$ and $z/R=1.6$. The velocity vectors (velocity times 180 sec) are plotted to scale on the picture. The domain size is 290×210 cm. The sphere is moving from right to left.

$Nt=2000$). In each layer, the displacements of the fluorescent spots were determined by an image processing technique. After correction for residual movements and after interpolation by a spline function, we obtain the velocity field, an example of which is shown in Fig. 6 for $Nt=750$, $F=3$, and $z/R=1.6$, which allows to compute the vorticity field. An example of the vorticity fields in different layers is presented in Fig. 7 for $Nt=750$, $F=3$, and $z/R=1.6$, 2.4, 3.2, 4, the domain size being approximately $90R\times 80R$. Figure 8 shows the vorticity fields in two symmetric layers $z/R=-2.4$ and $z/R=2.4$, for $Nt=750$ but $F=5$. One may notice that the two fields are not completely correlated with each other. The correlation coefficient turns out to be 0.6 in this case whereas, between the same levels at the same value of $Nt$, it equals 0.8 when $F=3$. This decorrelation indicates the onset of the multilayered regime. The motion has been determined in this way every $Nt=105$ from $Nt=-420$ up to $Nt\simeq 2000$ for $F=3$ and $F=5$. For each $F$ value, the experiment has been ran twice in order to possess two independent sets of data. The turbulent maximum velocity at $Nt=750$, $F=3$ and $z/R=0$ is of order $V\sim 0.1$ cm sec$^{-1}$ and the scale is $L\sim 40$ cm, giving a turbulent Reynolds number $\mathrm{Re}_t=VL/\nu\sim 400$ and a turbulent

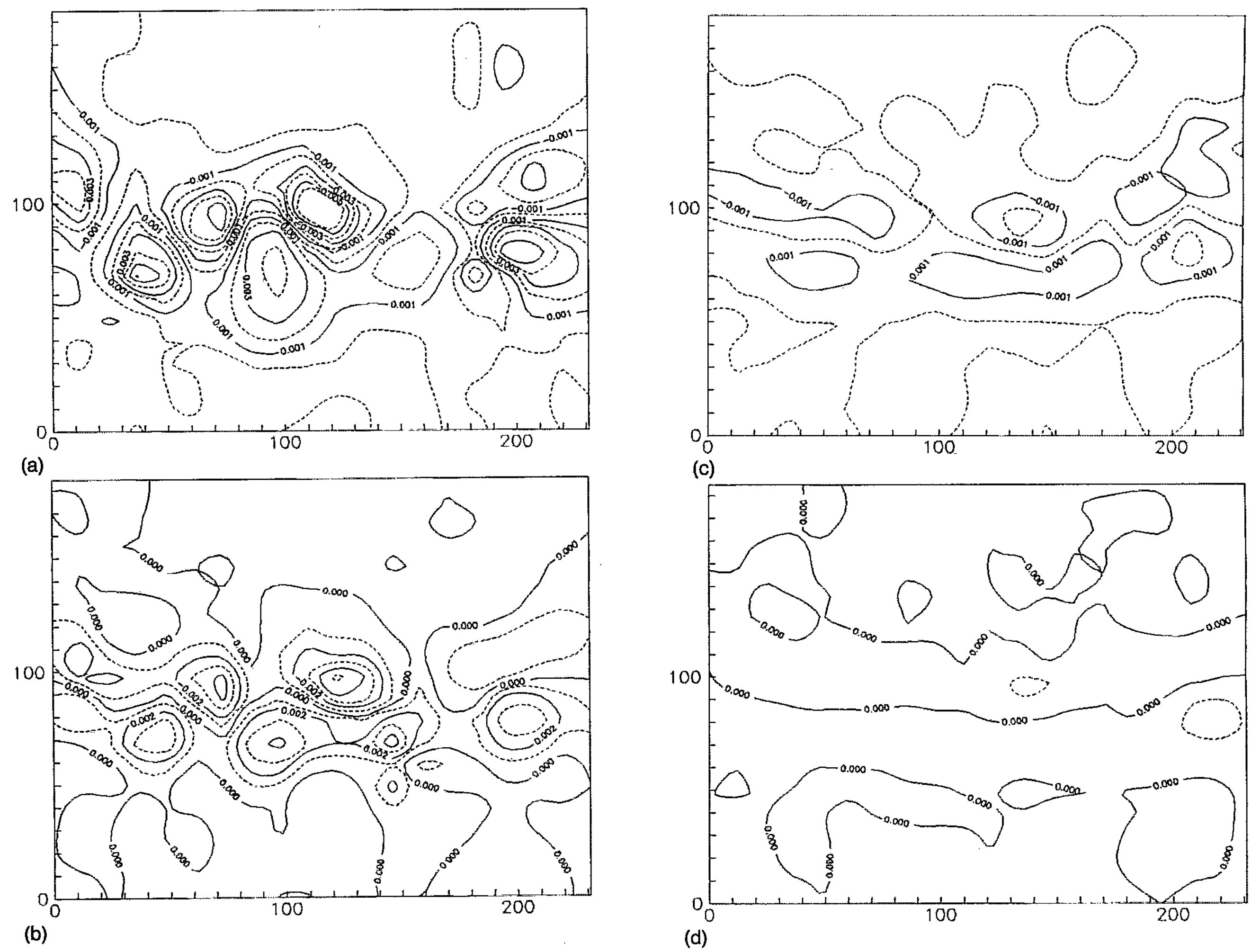


FIG. 7. Vertical vorticity field (in sec$^{-1}$) for $F=3$ at time $Nt=750$ at four horizontal planes distant from the sphere center by (a) $z/R=1.6$; (b) $z/R=2.4$; (c) $z/R=3.2$; (d) $z/R=4$. The sphere is moving from right to left.

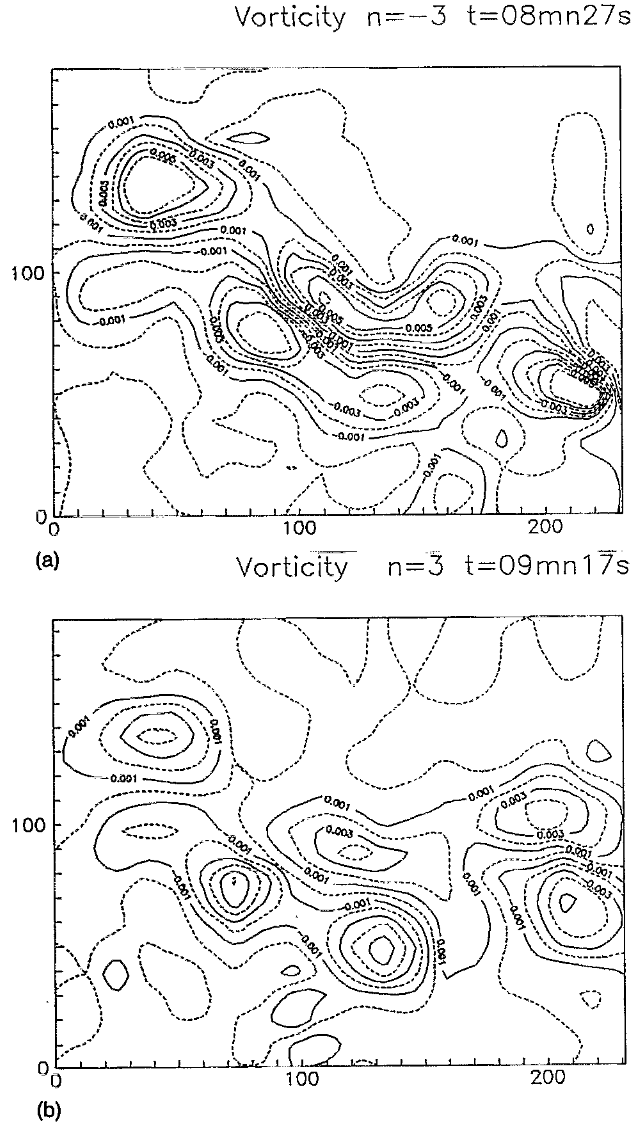


FIG. 8. Vertical vorticity field (in $\sec^{-1}$) showing the onset of the decorrelation (multilayered regime) for $F=5$ at time $Nt=750$ at two symmetric horizontal planes distant from the sphere center by (a) $z/R=-2.4$; (b) $z/R=2.4$. The sphere is moving from right to left.

Froude number $F_t = V/LN \sim 0.002$. At $z/R=2.4$ we have $V\sim 0.05$ cm $\sec^{-1}$, $\mathrm{Re}_t \sim 200$ and $F_t \sim 0.001$. In the case $F=5$, the turbulent maximum velocity at $Nt=750$ and $z/R=0$ is of order $V\sim 0.15$ cm $\sec^{-1}$ and the scale is $L\sim 50$ cm, giving a turbulent Reynolds number $\mathrm{Re}_t \sim 750$ and a turbulent Froude number $F_t \sim 0.0025$. At $Nt=750$ and $z/R=2.4$ we have $V\sim 0.05$ cm $\sec^{-1}$ giving $\mathrm{Re}_t \sim 250$ and $F_t \sim 0.001$.

The vertical line technique permitted to determine accurately the time of onset of the motion at a given position $z/R$. This time is defined as the instant when the vorticity passes the 0.001 $\sec^{-1}$ threshold corresponding to about 10% of the vorticity maximum at $z/R=2$, the precision on the vorticity measurement being of order 0.0002 $\sec^{-1}$. An alternative definition of the time of onset of the motion at a given position $z/R$ is given by the time at which the correlation of the vorticity field at the levels $z/R$ and 0 passes a 0.5 threshold value. This definition by correlation gives, in the present case, the same value as the definition by the maximum. This time, $t$, is plotted in Fig. 9 versus the distance of the layer below the sphere center for $F=3$ and $\mathrm{Re} \simeq 3920$ and for $F=5$ and $\mathrm{Re}=6530$. The time scale is kept in dimensional form because we do not know yet how the time scales with Froude number and Reynolds number. The model proposed below is an attempt in this direction. The viscous diffusion law (plotted on the Fig. 9) gives:

$$t_\nu = \left(\frac{z-z^0}{R}\right)^2 \frac{R^2}{\nu}, \qquad (1)$$

where $z^0$ is the initial vertical extension of the layer in motion. From observations of the close wake[11] $z^0$ equals nearly $R$ for $F=3$. Figure 9 shows that the motion diffuses vertically much more rapidly than one should expect from a viscous diffusion law. In Fig. 9 is also plotted the observed vertical diffusion time for $F=5$ and $\mathrm{Re}=6530$. The points for $F=5$ are below the precedent curve mainly because $z_0(\sim 1.5R)$ is larger than for $F=3$. The slope for $F=5$ is larger than for $F=3$, contrary to the model prediction developed in the following, certainly because the model do not take into account for the multilayered structure of the far wake ($F \geqslant 4.5$) which is believed to weaken the diffusion process. The most likely reason for this rapid vertical diffusion is secondary vertical motion in the far wake similar to Ekman pumping. The vortices are bounded at their ends by fluid at rest and the boundary layer thickness between the two is of order $\delta \sim \sqrt{\nu L/V}$. The equiva-

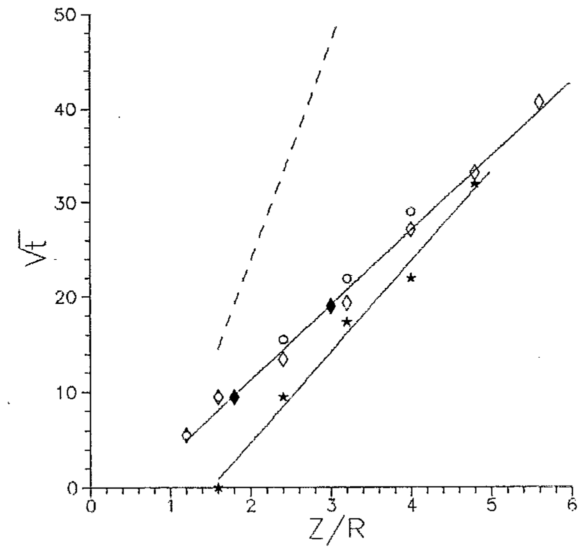


FIG. 9. Square root of the vertical diffusion time $t$ (in $\sec^{1/2}$) versus the distance from the wake center $z/R$. The theoretical purely viscous diffusion law is plotted by a dotted line (its slope equals $R\nu^{-1/2}$). The full lines are the interpolated experimental curve for $F=3$ and $\mathrm{Re}=3920$ and $F=5$ and $\mathrm{Re}=6530$. The observed time corresponds to the time when the motion begins to be measurable (vorticity higher than $10^{-3}$ $\sec^{-1}$) at the given layer $z/R$. ◊ and ○ correspond to $F=3$ and $\mathrm{Re}=3920$ (closed symbol are points obtained by the horizontal line technique and open symbol by the vertical line technique); ⋆ correspond to $F=5$ and $\mathrm{Re}=6530$, the points have been obtained by the vertical line technique. All measurements have been performed for $N=1.17$ rad×$\sec^{-1}$.

lent eddy viscosity associated with the secondary motion (Ekman layer type) would be $\nu_E \sim \delta V \sim \sqrt{\nu L V}$. By conservation of energy per unit length of the wake the evolution of the vortex can be expressed in term of an initial state by $V^2 LH = V_0^2 L_0 H_0$, thus giving for the eddy viscosity $\nu_E \sim \sqrt{\nu V_0 L_0 \sqrt{H_0 L/(H L_0)}}$, where $H_0/H = z_0/z$, $V_0$, and $L_0$ are the velocity and the width of the far wake, say at Nt=100. The expression for the diffusion time, normalized by the viscous diffusion time $t_\nu$ [Eq. (1)], then takes the form

$$\frac{t}{t_\nu} = \frac{1}{(\mathrm{Re}_t)_0^{1/2}} \left(\frac{z}{z_0}\right)^{1/4} \left(\frac{L_0}{L}\right)^{1/4}, \quad (2)$$

where $(\mathrm{Re}_t)_0 = V_0 L_0/\nu$. A numerical application gives typically a factor between 10 and 30 for $(\mathrm{Re}_t)_0^{1/2}$ depending on the way $V_0$ is estimated (maximum or averaged velocity at $z/R=0$ or at another level), which is in agreement with the results shown in Fig. 9.

## IV. CONCLUSION

The main objective of this paper is to demonstrate that the far wake of a sphere or, for that matter, any other object moving in a stratified fluid, diffuses vertically at a rate which cannot be explained by a purely viscous diffusion law. The time required for the far wake to double in vertical thickness is in the experiments typically one order of magnitude less than the viscous diffusion time. A simple model based on a concept of boundary layer renewal by secondary motion, due to centrifugal forces, can explain this enhanced diffusion. The time of diffusion would, according to this model, decrease as the inverse square root of the Reynolds number of the far wake based on the horizontal length scale. The horizontal spreading of the far wake has not been investigated because this horizontal diffusion is similar to the advance of a two-dimensional turbulent front with a spreading rate of order $V$.

A further important result peculiar to the sphere case, is that the far wake consists of a single layer when $F<4.5$ and when $F>4.5$ the far wake has a multilayered structure. The change from a single to multilayer structure is, however, not abrupt and $F=4.5$ is only a reference value.


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