Non-reversing modulated Taylor–Couette flows

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Abstract

We study the stability of Couette flow in the case in which the outer cylinder is held fixed and the speed of the inner cylinder oscillates harmonically with time around zero mean. We find that, if the amplitude of the oscillation is large enough, it is possible that the resulting time-dependent Taylor vortex flow rotates in a direction which does not depend on the direction of the azimuthal motion which drives it. The transition to this “non-reversing” Taylor vortex flow is examined as a function of frequency of modulation and radius ratio. We also find that “non-reversing” flow is present in the wavy modes regime.

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1. Introduction

In the original formulation of the Taylor–Couette problem, the fluid is contained between a fixed outer cylinder and a concentric inner cylinder which rotates at constant angular velocity $\Omega_1$. Our concern is the case in which $\Omega_1$ is not constant, but oscillates harmonically in time. We shall concentrate our attention…
on the simplest modulated Taylor–Couette configuration, for which

$$\Omega_1(t) = \Omega_{1a} \cos(\omega t),$$  

(1)

where $\Omega_{1a}$ is the dimensional modulation amplitude and the frequency of modulation $\omega$ is small enough that the damped viscous wave induced by the inner cylinder penetrates the fluid a distance $\delta_s = (2v/\omega)^{1/2}$ of the order of the gap between the cylinders, where $v$ is the kinematic viscosity. Previous related investigations have studied the high frequency limit (Barenghi et al., 1980), modulations around non-zero mean (Barenghi and Jones, 1989; Carmi and Tustaniwskyj, 1981; Hall, 1983; Kuhlmann et al., 1989; Riley and Lawrence, 1977; Walsh and Donnelly, 1988a,b), modulation of the inner cylinder in the axial direction (Marques and Lopez, 1997), and oscillatory flows between concentric spheres (Hollerbach et al., 2002; Zhang, 2002). In a recent paper (Youd et al., 2003) we have considered the case described by Eq. (1) and shown the existence (at radius ratio $\eta = 0.75$) of a transition to a new kind of axisymmetric time-modulated flow (called non-reversing Taylor vortex flow or NRTVF to distinguish it from reversing Taylor vortex flow or RTVF) in which the rotation direction of the Taylor vortices does not depend on the rotation direction of the inner cylinder.

The distinction between reversing and non-reversing flow is as follows: In the reversing case, the inner cylinder is rotating anti-clockwise (say) for the first part of the cycle; the vortices respond by rotating in their own particular direction. The inner cylinder then rotates in the opposite direction for the second part of the cycle, and the vortices respond by also changing their rotation direction.

In the non-reversing case the inner cylinder is again rotating anti-clockwise (say), and the vortices rotate in their own particular direction as before. However, when the inner cylinder reverses its rotation direction the vortices continue to rotate in the same direction as in the first half-cycle.

It is important to note that the existence of either flow is not a consequence of the initial seeding conditions we use in our numerical code. In the steady-state case, and using periodic boundary conditions, the bifurcation to a cellular flow (Taylor vortex flow) is a pitchfork, with one branch corresponding to cells which rotate in one direction, and the other branch corresponding to cells rotating in the opposite direction. The smooth transition to a cellular flow is due to some seeding “noise” which breaks the perfect pitchfork bifurcation symmetry. Changing the sign of this noise would select the opposite branch. The initial seeding noise is necessary to start up the flow and changing the sign of the initial condition merely changes the direction of rotation of the first cell. We cannot use the above argument in our modulated problem. The Reynolds number changes sign periodically, and the flow is a time-dependent solution of the Navier–Stokes equation which develops self-consistently, independently of some arbitrary noise. As shown in Figs. 11 and 14, depending on the parameters, this flow is different in the reversing and non-reversing cases. The situation is the same in a related time-dependent problem (Lopez and Marques, 2002).

The aim of this paper is to extend the initial investigation in the parameter space, and hence to assess the robustness of the new flow. We allow non-axisymmetric motion and cover a wide range of values of radius ratio and frequency.

2. Formulation of the problem

The governing equations are

$$\frac{\partial \tilde{v}}{\partial t} + \tilde{v} \cdot \nabla \tilde{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \tilde{v},$$  

(2)

Fig. 1. Steady Taylor–Couette flow. Critical wavenumber $c_0$ versus radius ratio $\eta$. (———) Our results, (------) Roberts’ results.

\[ \vec{\nabla} \cdot \vec{\nu} = 0, \]

where $\vec{\nu}$ is the velocity and $p$ is the pressure. The density, $\rho$, and the kinematic viscosity, $\nu$, are constant. We call $R_1$ and $R_2$, respectively, the inner and outer cylinder radii, and make the usual assumption that the cylinders have infinite length (infinite aspect ratio). Using cylindrical coordinates $(r, \phi, z)$ the no-slip boundary conditions are $v_r = v_\phi = v_z = 0$ at $r = R_2$, and $v_r = v_z = 0$, $v_\phi = R_1 \Omega_1(t)$ at $r = R_1$, where $\Omega_1(t)$ is given by Eq. (1). We make the equations dimensionless using the length scale $\delta = R_2 - R_1$ and the viscous time scale $\delta^2/\nu$. Eq. (1) is then expressed in terms of the Reynolds number

\[ Re_1(t) = R_{e_{\text{mod}}} \cos(\omega t), \]

where $R_{e_{\text{mod}}} = \Omega_{1a} R_1 \delta/\nu$ and now $t$ and $\omega$ are dimensionless. The other parameter of the problem is the radius ratio $\eta = R_1/R_2$ which measures the importance of curvature effects. We call $Re_{10} = \Omega_{10} R_1 \delta/\nu$ the Reynolds number, where $\Omega_{10}$ is the angular frequency which corresponds to the onset of Taylor vortex flow in the steady case.

Eqs. (2) and (3) are solved using a combination of second order accurate Crank–Nicolson and Adams–Bashforth methods. The velocity components are represented by potentials which are expanded over Fourier modes in the azimuthal and axial directions and over Chebyshev polynomials in the radial direction. The details of the numerical method are in Willis and Barenghi (2002). Here, it suffices to say that the code has been tested in the linear and nonlinear axisymmetric regime against published results of Barenghi (1991) and Jones (1985a) and in the wavy mode regime against the findings of King et al. (1984) and Marcus (1984). Our computed code is thus three-dimensional, that is to say, it allows for full resolution in the $r$, $\phi$, and $z$ directions.

3. Axisymmetric reversing and non-reversing flows

Calculations are performed at radius ratios of $\eta = 0.3, 0.5, 0.6, 0.7, 0.8$, and $0.9$. Figs. 1 and 2 show the dependence of the steady, critical (dimensionless) wavenumber $c_0$ and Reynolds number $Re_{10}$ on

\[ \begin{align*}
\Omega_{1a} & = 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \\
\end{align*} \]
radius ratio for the radius ratios explored. Roberts (1965) also calculated steady critical wavenumbers and Reynolds numbers and they are shown as dashed curves in these figures. The agreement between the results is excellent. There is a much stronger dependence of the wavenumber on the frequency in the modulated case, so the calculations were again all performed with variable \( \alpha \) to determine the critical axial wavenumber at each frequency. Non-axisymmetric calculations showed that the wavy modes always decayed and the resulting solution was axisymmetric. Our initial conditions consist of an approximate cellular flow of shape \( r^2(1-r)^2 \sin(\alpha z) \) with amplitude ranging from \( 10^{-10} \) to \( 10^{-3} \) as described in Willis and Barenghi (2002); we then integrate the equations off motion in time, until, after an initial transient, a settled oscillation is achieved.

Fig. 3 shows the critical wavenumber \( \alpha_c \) versus the frequency of modulation \( \omega \) for all radius ratios explored: the critical wavenumber remains nearly constant for frequencies greater than 5 for NRTVF,
and frequencies less than 3 for RTVF. As \( \omega \to 0 \) the RTVF branch tends to the steady critical wavenumber, \( \omega_{c0} \).

The appearance of NRTVF is identified by plotting the value of the radial component of the velocity, \( v_r \) at the outflow jet (\( z = \lambda/2 \), where \( \lambda \) is the wavelength of a pair of Taylor vortices) in the middle of the gap (\( r = (R_1 + R_2)/2 \)); in the case of NRTVF this value does not change sign during a cycle. The existence of RTVF is also made apparent by making contour plots of the radial velocity component over a cross-section in the \((r, z)\)-plane: the passage of an axial nodal line, \( v_r = 0 \), across the gap is the signature of a reversal.

Fig. 4 shows the critical Reynolds number \( Re_{\text{mod}} \) of the inner cylinder versus the frequency of modulation \( \omega \) for various radius ratios. The intersection point of the critical Reynolds number of reversing and non-reversing flow changes with \( \eta \). This change in the intersection point will be further explored below. The large difference between the critical Reynolds numbers at \( \eta = 0.8 \) and \( \eta = 0.9 \) is to be expected, and corresponds to the enhanced stability in the steady case as the narrow-gap limit is approached. Calculations were also done at \( \eta = 0.95 \) but at this radius ratio there was evidence of 3D motion. The nature of this 3D motion is explored in the next section. However, the axisymmetric calculations showed qualitatively similar results to the other radius ratios with \( Re_{\text{mod}} = 291.29 \) and 357.45 at \( \omega = 2 \) and 5, respectively, for RTVF, and \( Re_{\text{mod}} = 345.25 \) and 357.92 at \( \omega = 3 \) and 8, respectively, for NRTVF. The intersection occurs at \( \omega = 4.49 \) and \( Re_{\text{mod}} = 337.80 \).

It is apparent from Fig. 4 that, at each \( \eta \), there exists a critical frequency \( \omega_{c}^{*} \) at which RTVF and NRTVF set in at the same Reynolds number \( Re_{\text{mod}}^{*} \). Fig. 5 shows this critical frequency \( \omega_{c}^{*} \) as a function of \( \eta \). It appears that, as the radius ratio approaches the narrow-gap limit, the critical frequency at which both RTVF and NRTVF occur at the same Reynolds number increases.

The critical Reynolds number \( Re_{\text{mod}}^{*} \) of this intersection increases with radius ratio, as shown in Fig. 6. To make the comparison fair, at each \( \eta \) we have normalised \( Re_{\text{mod}}^{*} \) using the steady values \( Re_{10} \). For RTVF the corresponding critical wavenumbers \( \omega_{RTVF}^{*} \) (normalised by \( \omega_{c0} \)) are eventually constant \( (\omega_{RTVF}^{*}/\omega_{c0}) \) ranges from 0.858 at \( \eta = 0.3 \) to 0.842 at \( \eta = 0.9 \) whereas for NRTVF the critical wavenumbers \( \omega_{NRTVF}^{*} \) (again normalised by \( \omega_{c0} \)) increase with \( \eta \) \( (\omega_{NRTVF}^{*}/\omega_{c0} = 1.198 \) at \( \eta = 0.3 \) and \( =1.222 \) at \( \eta = 0.9 \).
The point at which RTVF and NRTVF have coincident critical Reynolds number $Re^\ast_{\text{mod}}$ is unusual because it has two critical wavenumbers $\sigma_{\text{RTVF}}^\ast$ and $\sigma_{\text{NRTVF}}^\ast$. Calculations at 5% above critical for each flow at the radius ratios explored show that the growth rate $\sigma$ is larger for NRTVF than RTVF. This suggests that the non-reversing flow is more likely to be favoured over the reversing flow.

4. Non-axisymmetric reversing and non-reversing flows

It is well known that, in the steady Taylor–Couette problem, if the Reynolds number $Re_1$ is sufficiently larger than $Re_{10}$, at some critical value which depends on the radius ratio, Taylor vortex flow (TVF) loses its azimuthal symmetry and becomes time-dependent wavy modes (WM). In the modulated Taylor–Couette problem the natural question to ask is thus whether or not the appearance of the wavy modes instability in the $\phi$-direction destroys the existence of non-reversing solutions. Is there a distinction between reversing
and non-reversing wavy modes (RWM and NRWM) similar to the distinction between RTVF and NRTVF? We are thus led to the study of what happens at large amplitudes of modulation, $Re_{mod}$.

Because of the large computational costs of solving the Navier–Stokes equations in the wavy modes regime, we do not attempt to determine the stability boundary of possible non-reversing wavy flows (which would require minimising $Re_{mod}$ as a function of $\xi$), but restrict our task to determining the existence (or non-existence) of non-reversing wavy flows.

It is important to note that, in the wavy modes regime, a change of sign of $v_r$ versus $r$ at fixed $z$ during the time evolution is not enough to guarantee that a reversal has taken place. This is because in the wavy modes regime the Taylor vortices oscillate up and down in the axial $z$ direction, so the value of $v_r$ at a fixed location in the $(r, z)$-plane can change sign even when a reversal is not taking place. To counteract this effect, we plot $v_r$ versus $r$ in a frame of reference moving with a particular vortex, tracking the location where $v_r$ is maximal/minimal within the entire Taylor vortex cell. If the flow is non-reversing then the profile of $v_r$ computed in this way is always positive/negative. If the flow is reversing then the profile of $v_r$ is positive for the first part of the cycle and negative for the second part (or vice-versa), with a smooth transition between the two as the nodal line crosses the gap.

The reason for tracking the maximal/minimal value of $v_r$ over the entire Taylor vortex cell is that if the axial position is fixed then it is still possible for $v_r$ to be positive in one part of the gap but negative in another, due to deformations of the Taylor vortex cells from a square to trapezoidal shape which occurs during the evolution. An example of this deformation can be seen in Fig. 7.

Fig. 7. Contour plot of radial velocity, $v_r$ for non-axisymmetric non-reversing flow. Note the trapezoidal deformation.

The figure shows a vertical cross-section of the radial velocity through a pair of Taylor vortices. The inner cylinder is on the left and the outer cylinder on the right. The plot extends to one wavelength, $\lambda = 2\pi/\xi$ in the axial direction. It can clearly be seen that at $z \approx 3\pi/2\xi$ and $z \approx \pi/2\xi$, where the radial velocity changes sign in the axial direction, it also changes sign in the radial direction. A plot of $v_r$ against time would then suggest that a reversal had occurred, which is not the case.

Having taken these necessary precautions, plots of $v_r$ versus $r$ computed as described can reveal if a flow is reversing or non-reversing. Fig. 8 shows a reversing flow for example at $\eta = 0.8$, $\omega = 4$, and $Re_{mod} = 250$. Here, we see the radial velocity versus radial position at nine different times when the actual reversal takes place. The curves should be read from the bottom (where $v_r < 0$) to the top (where $v_r > 0$) with time increasing in that direction. Initially, $v_r$ is negative everywhere across the gap. However, as time progresses $v_r$ becomes positive in parts of the gap (shown by the dashed parts of the curve in
Fig. 8. Radial velocity, $v_r$, computed by tracking the Taylor vortex as explained in the text, versus radial position, $r$, for non-axisymmetric reversing flow ($\eta = 0.8$, $\omega = 4$, $Re_{mod} = 250$). Each curve is plotted for a different time with $195.704 \leq t \leq 195.749$. The curves show $v_r < 0$, initially, and $v_r > 0$ after the nodal line has crossed the gap. The filled circles indicate the radial position where $v_r = 0$.

Fig. 9. Cross-sections of the radial velocity component for non-axisymmetric reversing flow. (a) at $t = 195.704$, (b) at $t = 195.726$, (c) at $t = 195.749$.

The whole reversal process is completed in only hundredths of a diffusion time. Fig. 10 shows a time-series of the radial velocity $v_r$ over two forcing periods for the non-axisymmetric reversing flow. The period of the forcing is $\tau = 2\pi/\omega$ which is 1.57 for the frequency of $\omega = 4$ shown. The reversing flow responds to this forcing synchronously and also has a period of 1.57. This synchronous response was also found in the axisymmetric regime (Youd et al., 2003), and is again in contrast to the (axisymmetric) results of Lopez and Marques (2002) who found that the reversing solutions were subharmonic with a period of twice that of the forcing, in modulated Taylor–Couette flow with a constantly rotating inner cylinder and sinusoidally modulated outer cylinder.
Fig. 10. Radial velocity, \( v_r \) (measured at the dimensionless position \( z = \pi/\alpha, r = (1 + \eta)/(1 - \eta) \)), versus \( t \), for non-axisymmetric reversing flow (\( \eta = 0.8, \omega = 4, R_{e\text{mod}} = 250 \)) over two forcing periods. Also shown is the Reynolds number \( R_{e1}(t) \). The period of the forcing is \( 2\pi/4 \approx 1.57 \) which the flow responds to synchronously. The time-step is \( \sim O(10^{-4}) \).

Fig. 11. Isosurfaces of helicity \( H = |\vec{v} \cdot (\vec{\nabla} \wedge \vec{v})| \) for non-axisymmetric reversing flow with parameters as in Fig. 8 shown over two axial periods. The predominant azimuthal mode is \( m = 1 \). The times of the plots are: (a) \( t = 41.012 \), (b) \( t = 41.425 \), (c) \( t = 41.489 \), (d) \( t = 41.886 \). The isosurface levels are taken at: (a) \( H \approx 12,000 \), (b) \( H \approx 5 \), (c) \( H \approx 30 \), (d) \( H \approx 6000 \).

Fig. 11 shows 3D perspective views of the non-axisymmetric reversing flow at four different times during a cycle. Values of the instantaneous Reynolds number and radial velocity at these times can be seen from the time-series plot of the radial velocity, Fig. 10. Shown are isosurfaces of helicity \( H = |\vec{v} \cdot (\vec{\nabla} \wedge \vec{v})| \) over two axial periods. It is apparent that the flow is predominantly \( m = 1 \), only looking the same after a full \( 360^\circ \) rotation about the axis. It was found in our previous paper (Yould et al., 2003) that there is a phase-lag between the change of direction of the inner cylinder and when the flow intensity is at a minimum due to the fact that the flow across the gap takes a certain amount of time to respond to the drive. Consequently, where Figs. 11(b) and (c) show the flow intensity at a minimum, the instantaneous Reynolds number is not close to zero. Helicity is a convenient quantity to represent the flow graphically, since it highlights the location of the Taylor vortex cells behind the layer of vorticity near the outer cylinder. It should be noted that there is a contribution from the helicity on the inner cylinder which is visible in the figures, but which does not obscure the cellular structure.
Fig. 12. Radial velocity, \( v_r \), versus radial position, \( r \), for non-axisymmetric NRTVF (\( \eta = 0.8, \omega = 5, Re_{\text{mod}} = 400 \)). Each curve is plotted for a different time with \( 171.92 \leq t \leq 172.00 \). The curves show that \( v_r < 0 \) for all times.

Fig. 13. Radial velocity, \( v_r \) (measured at the dimensionless position \( z = \pi/\alpha, r = (1 + \eta)/(1 - \eta) \)), versus \( t \), for non-axisymmetric non-reversing flow (\( \eta = 0.8, \omega = 5, Re_{\text{mod}} = 400 \)) over two forcing periods. Also shown is the Reynolds number \( Re_1(t) \). The period of the forcing is \( 2\pi/5 \approx 1.26 \) which the flow responds to synchronously. The time-step is \( \sim 6 \times 10^{-5} \).

Fig. 12 shows radial profiles as in Fig. 8 but now for a case which corresponds to non-reversing flow, at \( \eta = 0.8, \omega = 5, \) and \( Re_{\text{mod}} = 400 \). The profiles are plotted for one set of times where \( v_r \) is minimal within a vortex and close to zero (rather than over a full period). Again, reading the curves from bottom to top we can see that \( v_r \) is never positive. The radial velocity is initially strong and negative for earlier times where the inner cylinder is rotating in one particular direction, before becoming weak and negative when the cylinder is reversing its rotation direction, and finally again becoming strong and negative as the rotation rate of the cylinder increases in the opposite direction. This is repeated for all other times where \( v_r \) is close to zero.

Fig. 13 shows a time-series plot of the radial velocity \( v_r \) for the non-axisymmetric non-reversing flow again over two forcing periods. The period of the forcing in this case is \( 1.26 \), which the flow responds to synchronously with a period of 1.26. This result is in contrast to the results of the axisymmetric
Fig. 14. Isosurfaces of helicity $H = |\vec{v} \cdot (\vec{\nabla} \times \vec{v})|$ for non-axisymmetric non-reversing flow with parameters as in Fig. 12 shown over two axial periods. The predominant azimuthal mode is $m = 1$. The times of the plots are: (a) $t = 171.738$, (b) $t = 171.936$, (c) $t = 171.955$, (d) $t = 172.407$. The isosurface levels are taken at: (a) $H \approx 18,000$, (b) $H \approx 40$, (c) $H \approx 5$, (d) $H \approx 9000$.

calculations (Yould et al., 2003) where the non-reversing flow (NRTVF) responded harmonically with a period of half that of the forcing.

Isosurface plots similar to Fig. 11 are shown in Fig. 14. These perspective views reveal that the non-reversing flow is actually a spiral mode (and $m = 1$ is again the predominant azimuthal mode). With axial oscillations and constant rotation of the inner cylinder with a fixed outer cylinder (Marques and Lopez, 1997), and modulation of the outer cylinder with a constant rotation of the inner cylinder (Lopez and Marques, 2002), it was found that Neimark–Sacker bifurcations to a quasiperiodic solution were possible. (Lopez and Marques, 2002) found that the Neimark–Sacker bifurcation results in the formation of a family of spiral solutions, a left-hand spiral with azimuthal wavenumber $+m$, and a right-hand spiral with azimuthal wavenumber $-m$. They found that these spirals precess in the same sense as the cylinder rotation, but much slower, and also remarked that these solutions are generally quasiperiodic, but that strong resonances are found along the Neimark–Sacker curve. In our case, we have found that the spiral also precesses in the same sense as the inner cylinder rotation but at the same rate. Poincaré sections reveal that the flow is periodic, and a Fourier transform of the radial and axial velocities reveal that there is only one frequency in the solution — that of the forcing frequency. There is no evidence of quasiperiodic motion in our case.

Fig. 15 confirms that we are indeed dealing with a non-axisymmetric flow. The figure shows the contribution of each wavy mode component ($m = 0, 1, 2, \ldots$) to the total kinetic energy plotted versus time for a reversing flow (plots for non-reversing flows are similar). We can see that the kinetic energy of the wavy modes does not decay after the initial seeding. To aid clarity only the first three non-axisymmetric modes are shown, but the calculation includes spectral truncations of up to $M = 8$ in the azimuthal direction. A few words on the choice of radius ratio and truncation for the non-axisymmetric flows are relevant at this point. Jones (1985b) found that, in the steady case, as the narrow-gap limit is approached, it becomes easier to excite modes with higher and higher azimuthal wavenumber. This is also true in our modulated case and thus fully-resolved three-dimensional calculations in the narrow-gap limit $\eta \rightarrow 1$ require CPU times too large to be practical. On the other hand, if the radius ratio is too small — but not so small as to be out of the (steady) wavy regime, $\eta < 0.70$, say — then due to the large modulation amplitudes involved, the Reynolds number is not in the wavy regime for long enough and the wavy modes decay quickly. Consequently, finding a convenient regime to study in which wavy modes exist is a balancing act between azimuthal truncation at narrow gaps and exciting the azimuthal modes in wider gaps.
Fig. 15. Logarithm of the kinetic energy of the $m = 0$ (—–) axisymmetric mode and the $m = 1$ (–), 2 (-----), and 3 (--·--) azimuthal modes versus time.

5. Conclusion

In conclusion, our results show that, if the amplitude of the modulation is large enough to destabilise circular Couette flow, two classes of axisymmetric Taylor vortex flow are possible: reversing (RTVF) and non-reversing (NRTVF). The transition to NRTVF is robust, in the sense that it has qualitatively similar features at all values of radius ratios investigated ($0.3 \leq \eta \leq 0.9$). Furthermore, calculations in the wavy regime reveal that non-reversing solutions are not destroyed by the presence of azimuthal waves so we must distinguish between reversing and non-reversing wavy modes (RWM and NRWM), and the NRWM are actually spiral modes. It can be seen from Fig. 15 that the wavy modes dissipate far more quickly than the axisymmetric mode when the cylinder changes direction and so the reversing and non-reversing flows still persist in the wavy regime. The same effect (meridional motion in contrast to the azimuthal motion which drives it) has been recently observed also in spherical geometry (Zhang, 2002).

We conclude that there is a new class of non-reversing solutions of the modulated Taylor–Couette problem, in which the rotation in the $(r, z)$-plane does not depend on the rotation in the $\phi$ direction. This class of solutions occurs over a wide range of parameter space in terms of radius ratio and includes both axisymmetric and non-axisymmetric flows.

Further work will be concerned with finite aspect ratios to answer the question as to whether these non-reversing solutions are disturbed by the presence of end effects (Benjamin and Mullin, 1981; Cliffe et al., 1992).

References


