FUNDAMENTALS OF FLUID MECHANICS

MMI103

Antoine SELLIER

LadHyX. Ecole Polytechnique. FRANCE
sellier@ladhyx.polytechnique.fr

Eleven (11) 3-hour lessons made of course and exercices

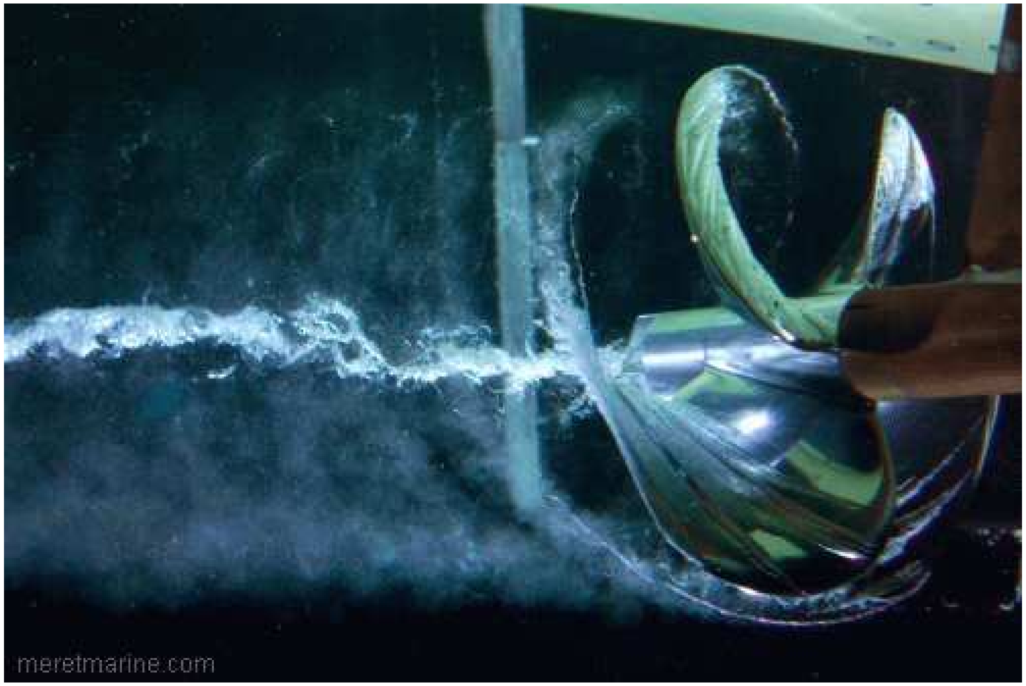
- 11, 18, 25 september
- 2, 9, 16 october
- A one-week break devoted to the first homework (HW1)
- 7, 13, 20, 27 november
- 4 december
- 18 december: written 2-hour or 3-hour exam (WE)

Four (4) additional lessons and one additional homework (HW2) delivered by another Professor

- taking place in January 2026 or February 2026
- A second homework (HW2) bearing on these lessons
- $N = HW1/4 + WE/2 + HW2/4$ with all HW1, WE and HW2 evaluated using a 0-20 scale

INCOMPRESSIBLE FLOW (constant density)

- Liquid, Hydrodynamics



Cavitation on the blade of a propeller (water, experimental observation)

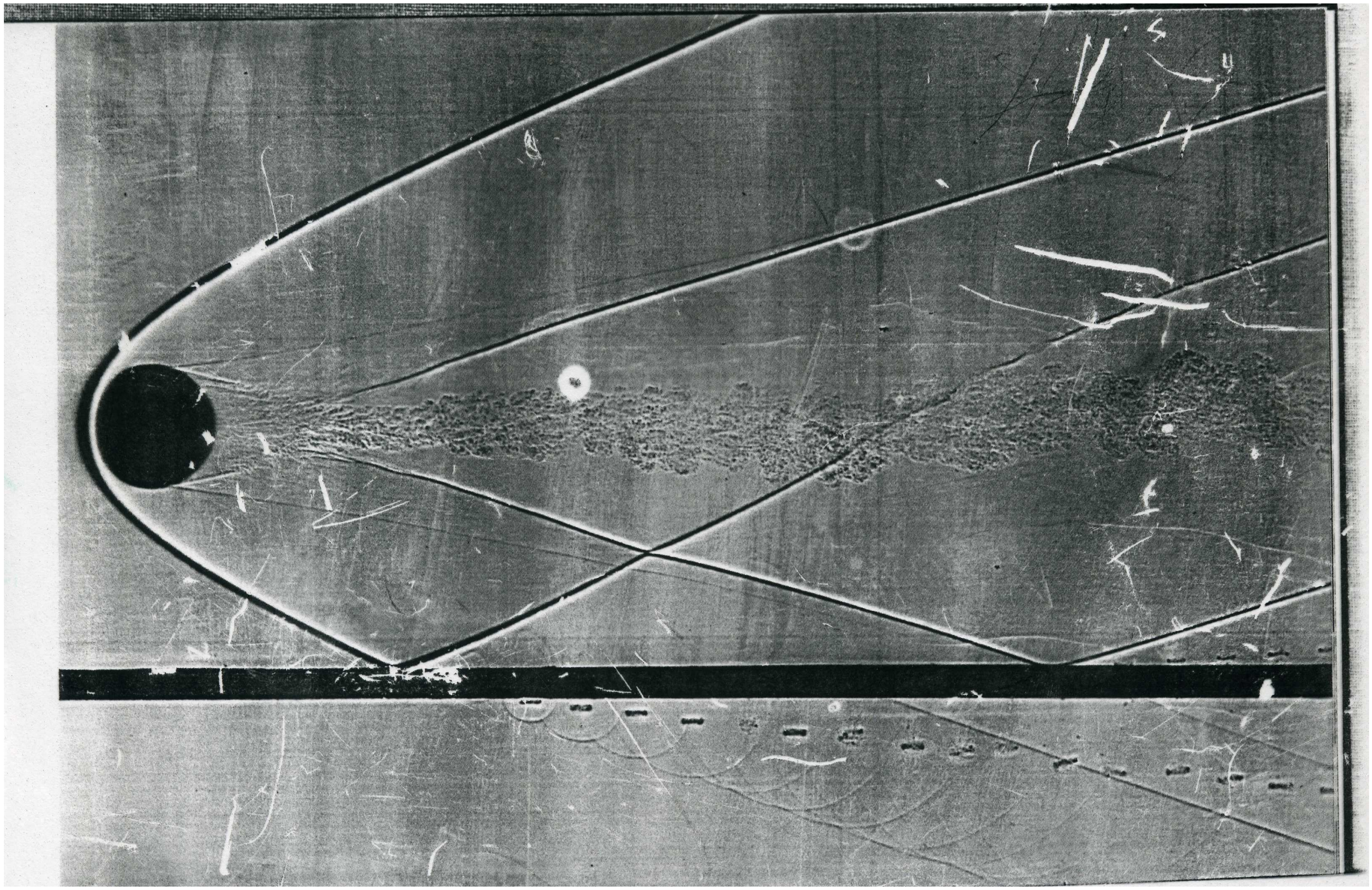
Production of bubbles, abrasion and visualization by cavitation

- Champagne! Role played by the bubbles? efficient mixing!

4. Les écoulements dans une flûte diffèrent selon que l'effervescence est naturelle *(a)* ou a lieu dans un verre gravé, ici par un anneau juste au-dessus du pied *(b)*. Dans cette même flûte gravée, une vingtaine de minutes après le versement, l'effervescence est moins intense *(c)*.

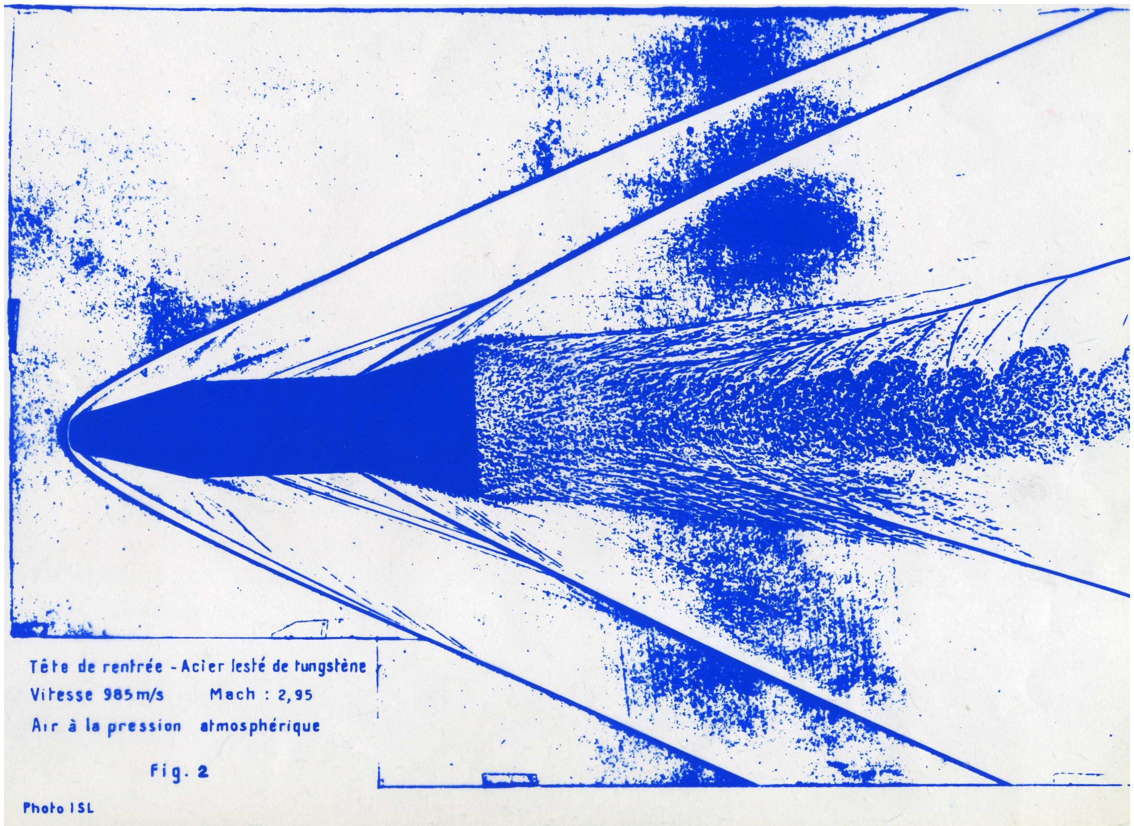
COMPRESSIBLE FLOW (non-constant density)

- acoustics, shock waves



COMPRESSIBLE FLOW

- When? "high velocity"
- Experimental observations



Lesson 1

MODELISATION

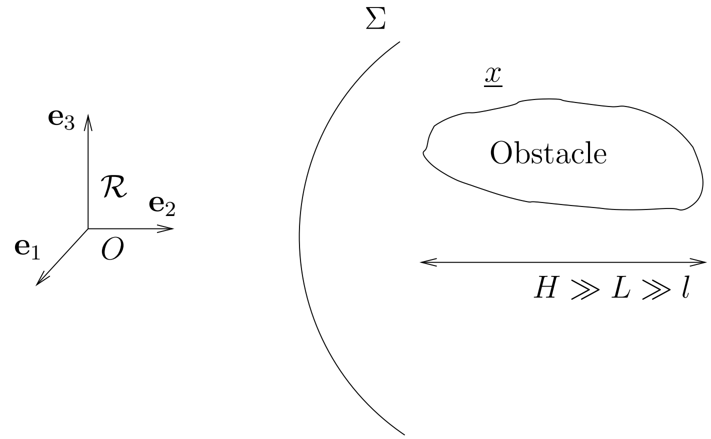
- Continuous medium. Eulerian description. Material derivative
- General problem for a fluid flow

FUNDAMENTAL GLOBAL CONSERVATION LAW

- Case of a steady domain
- Case of a moving domain. Summary
- Illustrating example: the mass conservation law

APPLICATION OF THE MASS CONSERVATION

CONTINUOUS MEDIUM

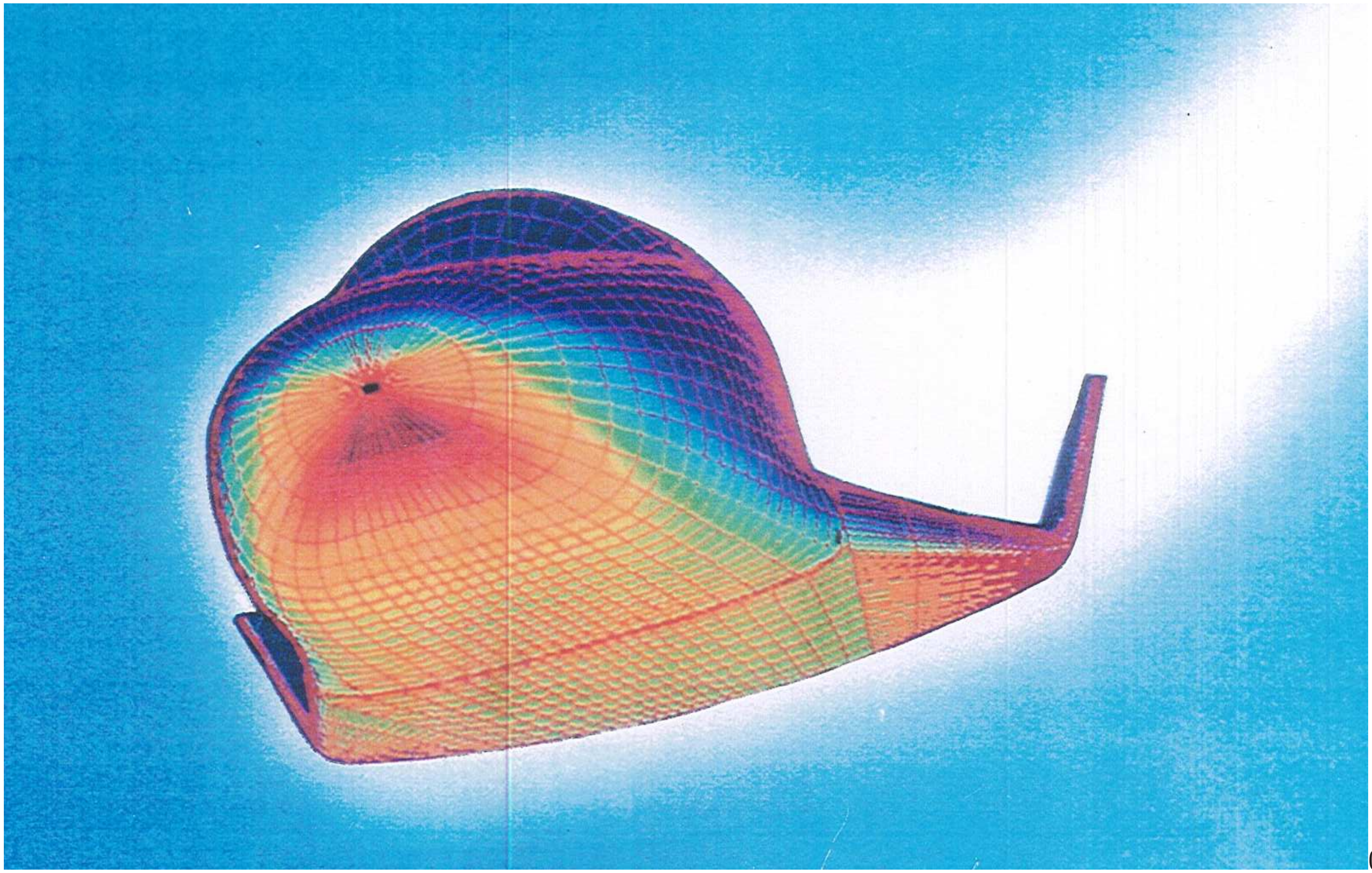


- H : body length scale
- l : molecules free space
- Knudsen number

$$Kn = l/H$$

The Knudsen number is not always small!

- If $Kn \ll 1$ then continuous medium
- Otherwise, not a continuous medium and statistical physics for a dilute medium



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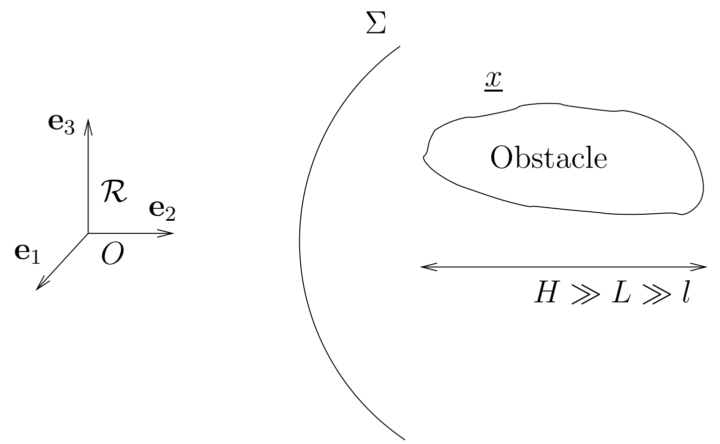
Space shuttle with size L

The free path l increases with the shuttle heigth!

CONTINUOUS MEDIUM



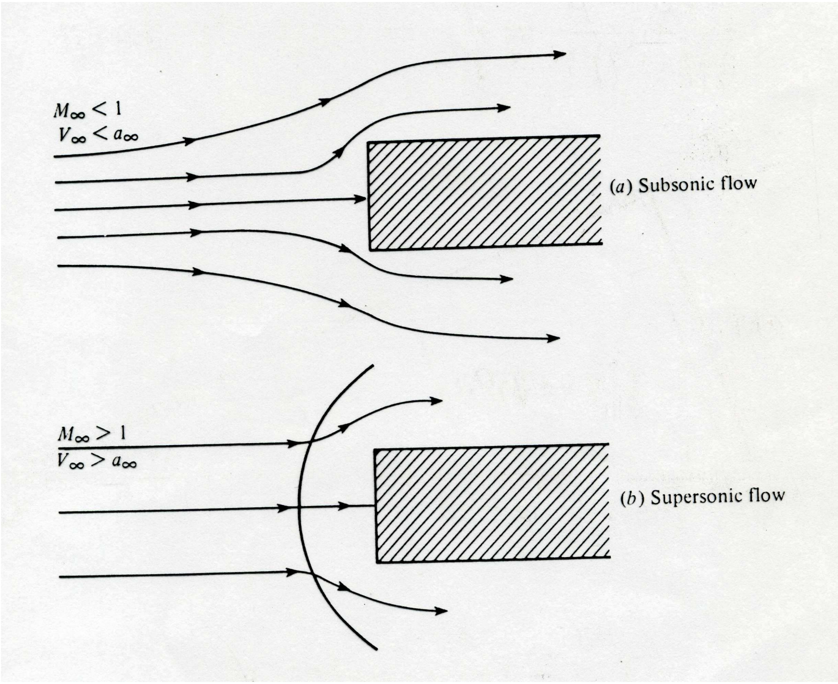
- Here $Kn \ll 1$ so that $l \ll L$
- Averaged quantities over domains (cubes) with typical length ϵ taking $l \ll \epsilon \ll L$
- Averaged quantity=macroscopic quantity g
- g can be a thermodynamic variable $\rho, p, T, ...$ or the the medium velocity $\underline{u}$
- Thermodynamic equilibrium. The medium usual equation-of-state holds

POTENTIAL DISCONTINUITIES AT THE SURFACE $\Sigma(t)$

- $\Sigma(t)$ can move at its own velocity but is INERT

Eulerian description

• physical quantity $\mathcal{G}$: associated to the macroscopic Eulerian field $g(\underline{x}, t)$



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• g can be the absolute température T, the density ρ, the pressure p, any Cartesian velocity component $u_i = \underline{u} \cdot \underline{e}_i$

Eulerian description

- Eulerian velocity field: $\underline{u}(\underline{x}, t)$
- flow streamlines at given time t

$$\frac{dx_1}{u_1(\underline{x}, t)} = \frac{dx_2}{u_2(\underline{x}, t)} = \frac{dx_3}{u_3(\underline{x}, t)}$$

- trajectory of a fluid particule

$$\underline{x} = \underline{X} = X_i \underline{e}_i \quad \text{à } t_0$$

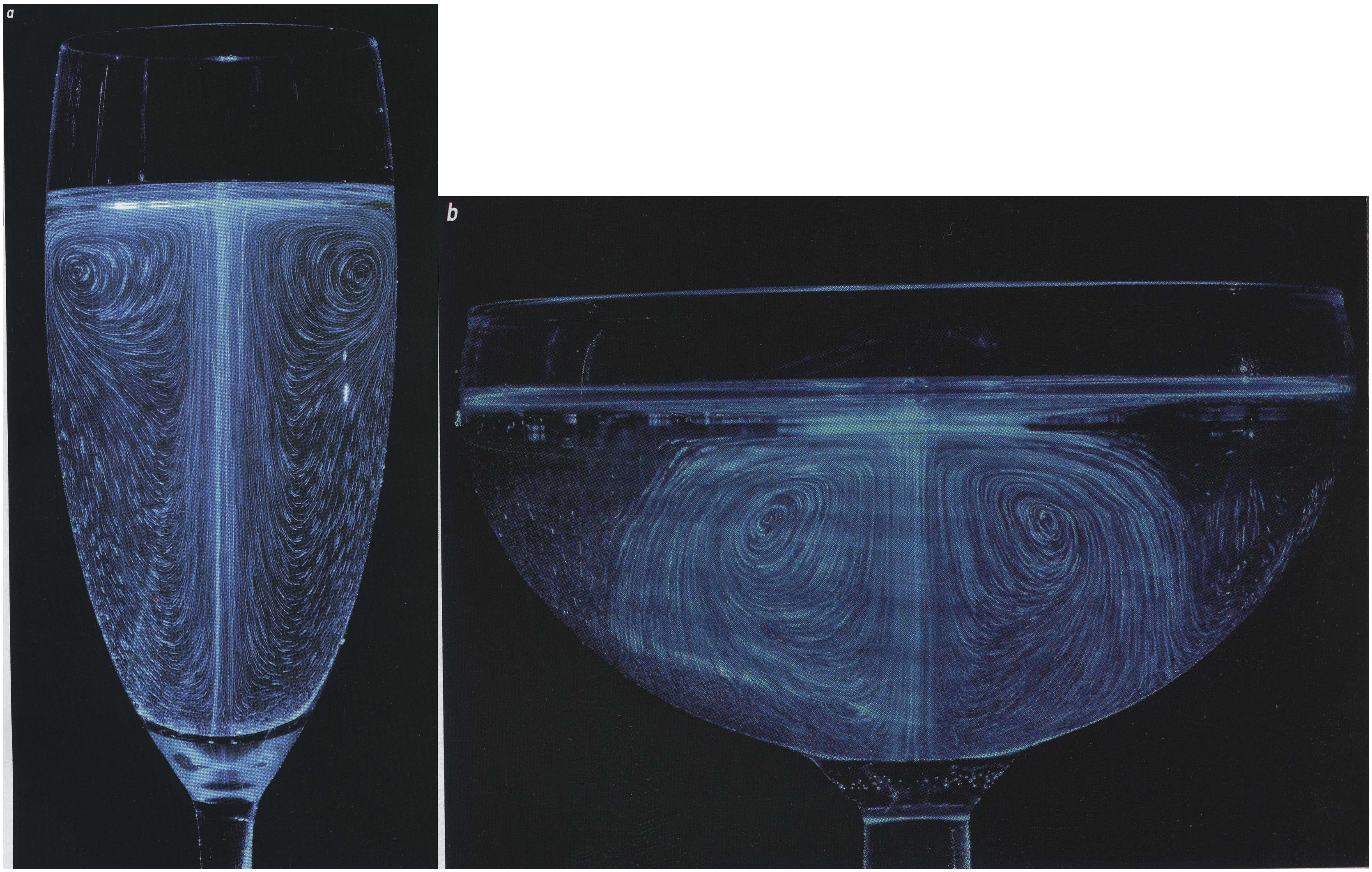
$$\frac{dx_i}{dt} = u_i(\underline{x}, t)$$

- Steady flow?

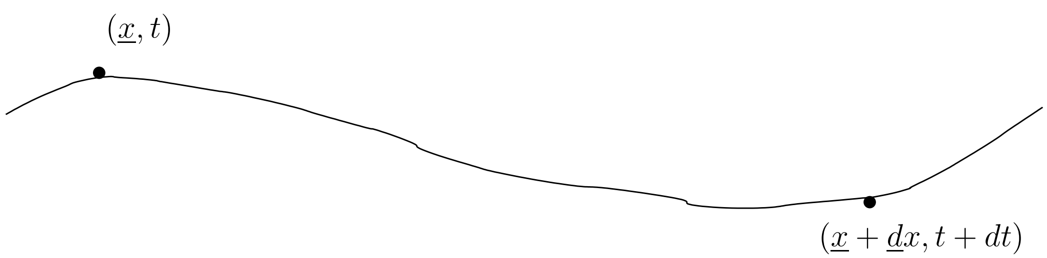
A flow the Eulerian description of which solely depends on $\underline{x}$, i. e. $g(\underline{x}, t) = g(\underline{x})$ whatever the flow physical quantity $\mathcal{G}$.

Then, streamlines and trajectories are the same lines

Streamlines at given time t for an unsteady flow



Material derivative



$\mathcal{G}$ described by its Eulerian field $g(\underline{x}, t)$

Tracking in time the same fluid particule: $\underline{dx} = \underline{u}(\underline{x}, t)dt$

$$\frac{dg}{dt} = \lim_{dt \to 0} \frac{g[\underline{x} + \underline{u}(\underline{x}, t)dt, t + dt] - g(\underline{x}, t)}{dt}$$

$$\frac{dg}{dt} = \frac{\partial g}{\partial t} + \underline{grad}[g].\underline{u}$$

Using orthogonal Cartesian coordinates

$$\underline{grad}[g] = \frac{\partial g}{\partial x_1}\underline{e}_1 + \frac{\partial g}{\partial x_2}\underline{e}_2 + \frac{\partial g}{\partial x_3}\underline{e}_3 = \frac{\partial g}{\partial x_i}\underline{e}_i$$

Acceleration?

$$\underline{\gamma} = \frac{d\underline{u}}{dt}, \qquad \underline{\gamma}.\underline{e}_i = \frac{\partial u_i}{\partial t} + u_j u_{i,j}$$

Fluid density $\rho(\underline{x}, t)$?

$$\frac{d\rho}{dt} = 0 : \quad \text{incompressible}$$

Not necessarily $\rho = cste$ everywhere!

Homogeneous fluid when $\rho = cste$ everywhere

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APPLICATION OF THE MASS CONSERVATION

GENERAL PROBLEM?

UNKNOWN FIELDS

- Eulerian description of the fluid motion $\underline{u}(\underline{x}, t)$
- Due to the fluid equation of state two variables $\rho(\underline{x}, t)$, $p(\underline{x}, t)$

DATA

- initial conditions at (t_0)
- applied fields: body forces, gravity,...
- boundary conditions dependent of the fluid nature/behaviour (see later
- fluid nature described by its equation of state such as $p = p(\rho, T)$
- fluid flow rheology described by a law (see later)

FUNDAMENTAL PHYSICAL LAWS TO BE APPLIED TO THE FLOW

- As previously mentioned: local thermodynamic equilibrium
- Fundamental laws of physics and thermodynamics

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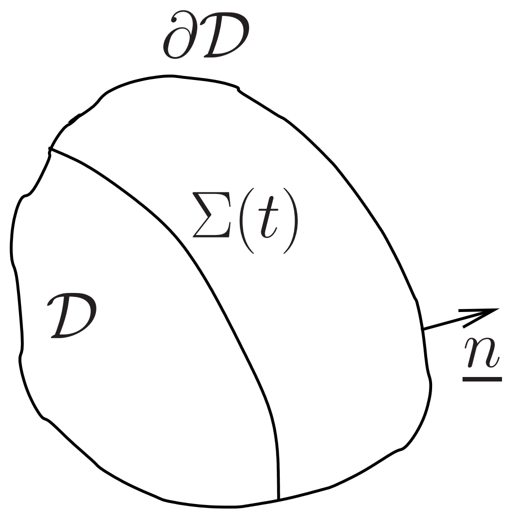
General global conservation law for a steady domain



Steady closed domain $\mathcal{D}$ with boundary $\partial\mathcal{D}$.

$\Sigma(t)$ is INERT for the quantity $\mathcal{F}$

$$\mathcal{F}(t) = \int_{\mathcal{D}} F(\underline{x}, t) d\Omega$$

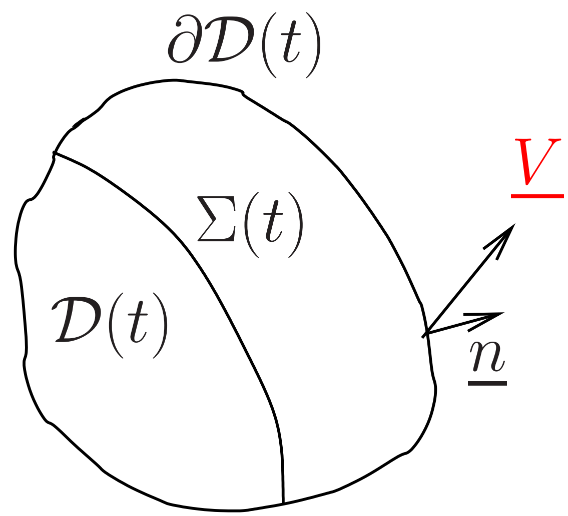
Form taken by a global conservation law

$$\frac{\delta}{\delta t} \int_{\mathcal{D}} F(\underline{x}, t) d\Omega = \int_{\mathcal{D}} P_F(\underline{x}, t) d\Omega - \int_{\partial\mathcal{D}} (F\underline{u} - \underline{A}).\underline{n} da$$

Material derivative of a volume integral?

Closed unsteady domain $\mathcal{D}(t)$ moving at the velocity field $\underline{V}$

$$\int_{\mathcal{D}(t)} F(\underline{x},t)d\Omega, \qquad \frac{\delta}{\delta t_{\underline{V}}}\int_{\mathcal{D}(t)} F(\underline{x},t)d\Omega?$$



If $\underline{V}$ and F are piecewise continuous

$$\frac{\delta}{\delta t_{\underline{V}}}\int_{\mathcal{D}(t)} F(\underline{x},t)d\Omega = \frac{\delta}{\delta t}\int_{\mathcal{D}} F(\underline{x},t)d\Omega + \int_{\partial\mathcal{D}(t)} F\underline{V}.\underline{n}da$$

OBTAINED EQUIVALENT FORMS TAKEN BY A GLOBAL CONSERVATION LAW

General case of a moving domain

$$\frac{\delta}{\delta t_{\underline{V}}}\int_{\mathcal{D}(t)} F(\underline{x},t)d\Omega = \int_{\mathcal{D}} P_F(\underline{x},t)d\Omega - \int_{\partial\mathcal{D}} [F(\underline{u}-\underline{V}) - \underline{A}].\underline{n}da$$

Steady domain

$$\frac{\delta}{\delta t}\int_{\mathcal{D}} F(\underline{x},t)d\Omega = \int_{\mathcal{D}} P_F(\underline{x},t)d\Omega - \int_{\partial\mathcal{D}} (F\underline{u} - \underline{A}).\underline{n}da$$

Material domain

$$\frac{d}{dt}\int_{\mathcal{D}(t)} F(\underline{x},t)d\Omega = \int_{\mathcal{D}(t)} P_F(\underline{x},t)d\Omega + \int_{\partial\mathcal{D}(t)} \underline{A}.\underline{n}da$$

Hold even in presence of a surface of discontinuities $\Sigma(t)$
INERT for the quantity $\mathcal{F}$

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GLOBAL MASS CONSERVATION

- System={selected fluid particles}

This system occupies the domain $\mathcal{D}(t)$ at time t

- Global mass conservation reads

$$\frac{d}{dt}\int_{\mathcal{D}(t)} \rho d\Omega = 0$$

Global mass conservation for a steady domain $\mathcal{D}$ reads

$$\frac{\delta}{\delta t}\int_{\mathcal{D}} \rho(\underline{x}, t) d\Omega = -\int_{\partial\mathcal{D}} \rho \underline{u}.\underline{n} da$$

NEXT WEEK

Lesson 2. Thursday 18th september, 9h00-12h15