Damping by branching: a bioinspiration from trees

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Abstract
Man-made slender structures are known to be sensitive to high levels of vibration due to their
flexibility which often cause irreversible damage. In nature, trees repeatedly endure large
amplitudes of motion, mostly caused by strong climatic events, yet with minor or no damage
in most cases. A new damping mechanism inspired by the architecture of trees is identified
here and characterized in the simplest tree-like structure, a Y-shaped branched structure.
Through analytical and numerical analyses of a simple two-degree-of-freedom model,
branching is shown to be the key ingredient in this protective mechanism that we call
damping-by-branching. It originates in the geometrical nonlinearities so that it is specifically
efficient to damp out large amplitudes of motion. A more realistic model, using flexible beam
approximation, shows that the mechanism is robust. Finally, two bioinspired architectures are
analyzed, showing significant levels of damping achieved via branching with typically 30% of
the energy being dissipated in one oscillation. This concept of damping-by-branching is of
simple practical use in the design of very slender and flexible structures subjected to extreme
dynamical loadings.

1. Introduction

Vibrations in man-made structures are a central problem in mechanical engineering (Den Hartog 2007). They may result from external excitations such as wind, impacts or earthquakes, or from internal excitations, such as a flow or moving parts. Their consequences are numerous in terms of functionality losses due to wear, fatigue or noise, to cite a few. We distinguish here between low and high levels of vibration. The former, where displacements are small in comparison with the characteristic size of the structure, may induce some of the long-term above-cited consequences. The latter generally cause short-term failures and irreversible damage to the structure by fracture or plastic deformation (Collins 1993). These large amplitudes of vibration may be particularly expected in slender structures, or assemblages of them, due to their high flexibility.

In the most general framework of vibration analysis, the amplitude of motion results, on one hand, from the characteristics of the loading, and on the other hand, from the characteristics of the structure in terms of inertia, stiffness and damping (Humar 2002). Damping here refers to the

capability of the structure to dissipate mechanical energy, whatever the physical mechanism involved (viscoelasticity, friction or interaction with a fluid). A high level of damping in a structure is a standard way to reduce amplitudes of motion. This is generally achieved with passive techniques, such as the classical addition of dampers (Krenk 2000), tuned mass-damper systems (Den Hartog 2007, p 119) or with active or semi-active means such as piezoelectric materials, magnetorheological fluids, shape memory alloys or even simple hydraulic actuators in feedback or feedforward systems (Preumont 2002). All these approaches have limits in terms of cost or maintenance but more particularly in terms of their range of acceptable deformations or displacements since they are not specifically designed to damp out large-amplitude vibrations. Efficient and specific damping for extreme dynamical loadings are of particular interest for slender and flexible structures such as antennas which may encounter large flow-induced amplitudes of vibration during such events (Païdoussis *et al* 2011).

Nature may give insights into highly efficient mechanical solutions in vibration problems, for instance, in shock-absorbing devices (Yoon and Park 2011). Interestingly, slender
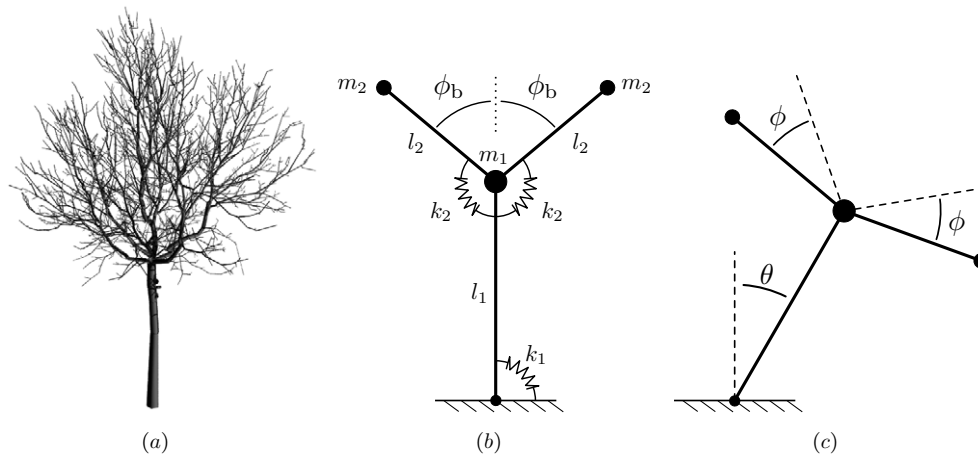
Figure 1. Branched geometries. (*a*) The walnut tree architecture analyzed by Rodriguez *et al* (2008). (*b*) and (*c*) Our Y-shaped spring-mass model of an elementary branched tree-like structure.

structures are ubiquitous in nature, particularly in plants. Most of these plants are regularly subjected to natural flow excitations by wind or current causing vibrations (de Langre 2008). Although these vibrations contribute to some biological functions such as in seed or pollen dispersion, extreme events such as storms may cause dangerous large amplitudes of motion (Niklas 1992). Therefore, in areas where intense flows are common, plants are likely to possess efficient and specific strategies to damp out vibrations of large potentially dangerous amplitudes.

From a biomimetic point of view, the dynamical behavior of trees, which has been extensively studied, is certainly a possible source of inspiration. Scannell (1984) hinted that trees might possess a 'qualitative mechanical design principle [...] beneficial to the tree's survivability in conditions of strong atmospheric turbulence'. Niklas (1992, p 183) noted that 'experiments indicate that branching [...] dampens natural frequencies of vibration'. At this point it is necessary to clarify what is generally agreed to cause damping in trees. Firstly, the constitutive material, wood, is known to have inherent viscoelastic behavior causing dissipation, this itself has been the source of bioinspired material (Spatz *et al* 2004). Secondly, the aeroelastic interaction with the surrounding air causes forces in the opposite direction to the local velocity in the tree, thereby causing a strongly amplitude-dependent dissipation (Blevins 1990). Finally, when considering the overall motion of the tree by bending of the trunk, another mechanism is often described as 'structural damping' (Brüchert *et al* 2003, Speck and Spatz 2004, James *et al* 2006, Moore and Maguire 2008). This third mechanism refers to the possible transfer of mechanical energy from the trunk to the branches, where it will be eventually dissipated by the two aforementioned aeroelastic and viscoelastic damping mechanisms (Sellier and Fourcaud 2009). But still it is not clear if this energy transfer mechanism is amplitude dependent or not. By modeling the tree branches as coupled tuned-mass-damper systems, Spatz *et al* (2006) have shown that the frequency tuning of the branches with the trunk plays a key role in this energy transfer mechanism in trees. This model shows, by definition, a purely linear energy transfer mechanism between parts of the

whole structure so that it is not amplitude dependent. More recently, Rodriguez *et al* (2008) analyzed the architecture of an actual walnut tree using finite element models, figure 1(*a*), and have shown that the modal frequencies are close and that the modal shapes are strongly localized in the architecture. The former characteristic is classically favorable to nonlinear modal energy exchanges in dynamical structures and associated with the latter would be consistent with an amplitude-dependent energy transfers from the trunk to the branches.

In order to develop strategies for bioinspired designs of slender structures including an efficient damping effect specific to large amplitudes, it is crucial to clarify the nonlinear mechanism involved in the energy transfer that many authors invoke. The aim of this paper is therefore to identify and characterize the elementary mechanism causing nonlinear modal energy transfer and damping in a branched structure specifically in the case of large-amplitude motions.

For this purpose, we first consider the simplest model of a branched dynamical system in section 2, a spring-mass model of a Y-shape. Section 3 shows, using a beam-finite-element model, that the main results of the previous section are also valid for a more realistic continuous structure of a Y-shape. Based on these results, two illustrative designs of bioinspired slender structures exhibiting efficient damping-by-branching are proposed in section 4. The generality and possible extensions of our approach are discussed in section 5.

2. Lumped-parameter model of a Y-shape

In order to reduce the dynamics of a branched structure to its simplest possible features, we treat the case of a spring-mass model of a Y-shape consisting of a trunk and two branches. Since we are interested in the branching effect, viscous damping is introduced in the branches only. The equations of motion are written with dimensionless variables and the dynamics is studied with an emphasis on the damping of the whole structure.

2.1. Model

The model consists of three massless rigid bars linked by rotational springs and supporting three masses, figure 1(*b*). The first bar, mimicking a trunk of length l_1, is linked to the ground by a rotational spring k_1 and supports a mass m_1. The branches are two symmetrical bars of length l_2, each forming an angle ϕ_b with respect to the trunk axis. Each branch is linked to the tip of the trunk by a rotational spring k_2, and supports a mass m_2. The motion of the trunk is defined by the angle θ, and we consider only the symmetrical motion of the branches defined by the angle ϕ, figure 1(*c*). Note that this restriction is made in order to simplify the following dynamical analysis of a two-degrees-of-freedom model. It has a small impact on the results described in this section compared to a full three-degrees-of-freedom model where each branch has an independent angle of motion. Moreover, this choice will be validated in the following sections for much more complex models which have no such restrictions. The kinetic energy is the sum of the kinetic energy of each mass:

$$T = \tfrac{1}{2}\big[\big(m_1l_1{}^2 + 2m_2\big(l_1{}^2 + 2l_1l_2\cos(\phi_b+\phi)+l_2{}^2\big)\big)\dot\theta^2 \\ + 2m_2l_2{}^2\dot\phi^2\big]. \tag{1}$$

The potential energy is the sum of the potential energy of each spring,

$$V = \tfrac{1}{2}(k_1\theta^2 + 2k_2\phi^2). \tag{2}$$

The equations of motion are derived using T and V in the classical framework of Lagrangian dynamics (Humar 2002). They read

$$J_\theta\ddot\theta + k_1\theta = 4m_2l_1l_2[\dot\theta\dot\phi\sin(\phi_b+\phi) - \ddot\theta J_\phi(\phi)], \\ 2m_2l_2{}^2\ddot\phi + 2k_2\phi = -2m_2l_1l_2\dot\theta^2\sin(\phi_b+\phi), \tag{3}$$

where

$$J_\theta = m_1l_1{}^2 + 2m_2\big(l_1{}^2 + 2l_1l_2\cos\phi_b + l_2{}^2\big), \tag{4}$$

and

$$J_\phi(\phi) = \cos(\phi_b+\phi) - \cos\phi_b. \tag{5}$$

The left-hand side of this system of equations represents two simple linear harmonic oscillators. Denoting the generalized displacement vector $[\theta, \phi]$, the two corresponding normal modes of the system are directly [1, 0] and [0, 1] since there is no linear coupling between θ and ϕ. The two modal angular frequencies are respectively

$$\omega_1{}^2 = \frac{k_1}{J_\theta} \qquad \text{and} \qquad \omega_2{}^2 = \frac{2k_2}{2m_2l_2{}^2}. \tag{5}$$

The first mode consists of motion involving θ only, and the second mode involving ϕ only. Therefore, in the following, they are referred to as the trunk mode and the branch mode, respectively. These two modes are coupled by the nonlinear terms of the right-hand side of (3), representing the geometric nonlinearities.

A dimensional analysis reveals the existence of four dimensionless parameters describing the dynamics of the model. We choose the dimensionless time $\tau = \omega_1 t$, the branching angle ϕ_b, the ratio of angular frequencies $\Omega =$ ω_2/ω_1, and the ratio Γ between the inertial terms of the branch mode and the trunk mode, multiplied by the length ratio l_1/l_2:

$$\Gamma = \frac{2m_2l_2{}^2}{J_\theta}\frac{l_1}{l_2} = \frac{2m_2l_1l_2}{J_\theta}. \tag{6}$$

The dynamics is described by the variables $\Theta(\tau) = \theta(t)\sqrt{l_1/l_2}$ and $\Phi(\tau) = \phi(t)$. As mentioned earlier, we introduce energy dissipation in the form of a viscous damping rate ξ_b in the branch mode only. The dimensionless equations of motion are

$$\ddot\Theta + \Theta = 2\Gamma[\dot\Theta\dot\Phi\sin(\phi_b+\Phi) - \ddot\Theta J_\phi(\phi)], \\ \ddot\Phi + 2\Omega\xi_b\dot\Phi + \Omega^2\Phi = -\dot\Theta^2\sin(\phi_b+\Phi). \tag{7}$$

The dimensionless total mechanical energy is

$$E(\tau) = \tfrac{1}{2}[(2\Gamma J_\phi(\phi)+1)\dot\Theta^2 + \Theta^2 + \Gamma(\dot\Phi^2 + \Omega^2\Phi^2)]. \tag{8}$$

Since the two modes are coupled by nonlinear terms, energy can be exchanged between them. In this case, the dissipation in the branch mode may damp the energy received from the trunk mode, resulting in an effective damping of the whole structure.

2.2. Damping criterion

In the following, we examine the free vibrations following an initial condition

$$[\Theta(0), \dot\Theta(0), \Phi(0), \dot\Phi(0)] = [\Theta_0, 0, 0, 0], \tag{9}$$

such that the energy is located in the undamped trunk mode only. This will allow us to easily demonstrate damping by nonlinear modal energy transfer, if any, since in a purely linear framework, energy would remain in the undamped trunk mode with no way of being dissipated. The amplitude of the initial condition, Θ_0, determines the initial energy $E(0) = E_0$, using (8). For the sake of clarity, the energy E is normalized so that the initial energy E_0 is 1 when $\Theta_0 = \pi/2$ corresponding to a horizontal trunk initial condition. Note that ground interaction is neglected here.

During free oscillations, a part of the energy transferred from the trunk mode to the branch mode is dissipated. The total energy decay over the first period of the trunk mode is $\Delta E = E_0 - E(2\pi)$ so that the effective damping rate of the whole structure can be defined as

$$\xi_{\text{eff}} = \frac{1}{4\pi}\frac{\Delta E}{E_0}. \tag{10}$$

The effective damping rate, ξ_{eff}, is commonly related to the quality factor Q by $Q = 1/(2\xi_{\text{eff}})$. Note that ξ_{eff} represents the dissipation of the whole structure and not that of the trunk mode. In fact, studying exclusively the trunk mode damping is not appropriate since energy transfer can be reciprocal from the branch mode to the trunk mode as well, as will be seen in figure 3. The total energy decay ΔE is given by the work of the damping term of the branch mode equation over one period of the trunk mode:

$$\Delta E = \frac{8}{\pi^2}\Gamma\int_0^{2\pi}2\Omega\xi_b\dot\Phi^2\,d\tau. \tag{11}$$

Here, the coefficient $8/\pi^2$ comes from the normalization chosen for E. We analyze now the effect of the initial energy E_0 and the design parameters ϕ_b, ξ_b, Ω and Γ on the effective damping, ξ_{eff}.

2.3. Energy transfer by internal resonance

We first consider a low initial energy level so that $\Theta_0 = \varepsilon$, where $\varepsilon \ll 1$ is a small parameter. The harmonic balance method (Nayfeh *et al* 1979) is used with the angles Θ and Φ developed as the power series of ε:

$$\Theta(\tau) = \varepsilon\Theta_1(\tau) + \varepsilon^2\Theta_2(\tau) + \cdots, \qquad (12)$$

$$\Phi(\tau) = \varepsilon\Phi_1(\tau) + \varepsilon^2\Phi_2(\tau) + \cdots. \qquad (13)$$

The initial condition (9) requires that

$$\Theta_1(0) = 1 \quad \text{and} \quad \Theta_2(0) = \Phi_1(0) = \Phi_2(0) = 0. \qquad (14)$$

Substituting (12) and (13) in the dynamical equations (7), and using (14), the first-order terms are

$$\Theta_1 = \cos\tau \quad \text{and} \quad \Phi_1 = 0. \qquad (15)$$

The second-order terms satisfy respectively

$$\Theta_2 = 0 \quad \text{and} \quad \ddot{\Phi}_2 + 2\Omega\xi_b\dot{\Phi}_2 + \Omega^2\Phi_2 = -\dot{\Theta}_1{}^2\sin\phi_b. \qquad (16)$$

Therefore, for small angles, (7) reduces to

$$\ddot{\Phi} + 2\Omega\xi_b\dot{\Phi} + \Omega^2\Phi = -\Theta_0^2\sin\phi_b\frac{(1 - \cos 2\tau)}{2}. \qquad (17)$$

This is the equation of a simple harmonic damped oscillator, driven by a harmonic force, that can be analytically solved (Humar 2002). A resonance exists at $\Omega = 2$; since Ω is the frequency ratio of the two modes, this is classically referred to as a 1:2 internal resonance (Nayfeh *et al* 1979). In the following, we will discuss the influence of Ω near this particular value. A general result for a forced damped oscillator is that the amplitude of motion is proportional to the amplitude of the driving force. As can be seen in (17), the amplitude of the driving force is proportional to $\Theta_0^2\sin\phi_b$, and therefore to $E_0\sin\phi_b$. The effective damping ξ_{eff}, defined by (10) and (11), can therefore be simply expressed as

$$\xi_{\text{eff}} = E_0\Gamma\sin^2\phi_b\bar{\xi}(\xi_b, \Omega). \qquad (18)$$

Remarkably, (18) shows that ξ_{eff} increases linearly with the initial energy E_0: such a nonlinear damping proportional to the energy is typical of an oscillator following the generic equation $\ddot{\Theta} + \kappa\dot{\Theta}^3 + \Theta = 0$ (Nayfeh *et al* 1979). Besides, (18) shows that ξ_{eff} is proportional to $\sin^2\phi_b$ so that the effective damping is maximal for a branching angle $\phi_b = \pi/2$, corresponding to a T-shaped structure. Conversely, for a non-branched structure, where $\phi_b = 0$ or π, the effective damping is zero. The effective damping is also proportional to the relative modal mass ratio Γ. The dependence of ξ_{eff} on Ω and ξ_b is shown in figure 2 as a contour map of the normalized effective damping $\bar{\xi}$, computed using mathematical symbolic software to solve (17) for Φ, and then using successively (11), (10) and (18) for $\bar{\xi}$.

As expected, we observe in figure 2 that there is no effective damping for $\xi_b = 0$ since mechanical energy cannot be dissipated in the structure. Interestingly, for any arbitrary small value of ξ_b, the effective damping is finite. In a purely linear framework, the effective damping would be zero for
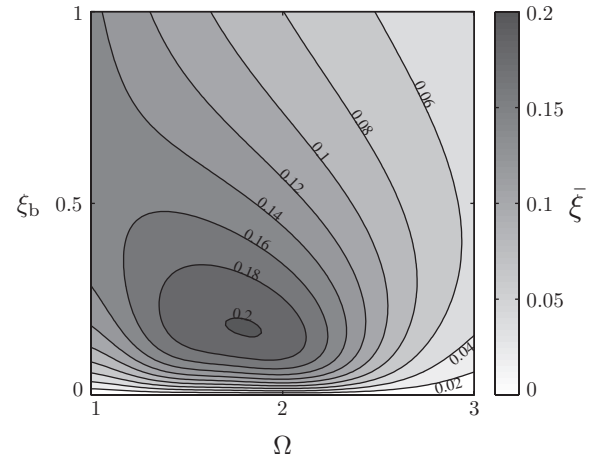

Figure 2. Normalized effective damping $\bar{\xi}$, (18), brought about by branching, as a function of the branch mode damping ξ_b, and the branch/trunk modal frequency ratio Ω. High damping is found near the 1:2 internal resonance, i.e. at $\Omega = 2$.

any value of ξ_b, since the total energy would be confined to the trunk mode, without any possible transfer to the branch mode where dissipation occurs. In other words, the effective damping is due to the geometric nonlinearities.

We observe that a significant level of damping is present over a wide range of parameter values. The effective damping shows a maximum for branch damping near 0.2 and frequency ratio near 2. Accordingly, the values $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$ will be used as a reference in the remainder of this paper.

2.4. Effects of the design parameters

In order to obtain the full dynamics and the corresponding effective damping at any energy level with an emphasis on the effects of the design parameters ϕ_b, ξ_b, Ω and Γ, the dynamical system (7) is now solved numerically by means of a fourth-order explicit Runge–Kutta temporal scheme.

As a typical example, figure 3 shows, for an initial energy level $E_0 = 1$, the evolution of the total energy E and of the modal energies

$$E_\Theta = \frac{4}{\pi^2}(\dot{\Theta}^2 + \Theta^2) \quad \text{and} \quad E_\Phi = \frac{4}{\pi^2}\Gamma(\dot{\Phi}^2 + \Omega^2\Phi^2). \qquad (19)$$

Note that the total energy E, (8), is the sum of E_Θ, E_Φ and a nonlinear energy term. The energy exchange between the two modes is clearly shown. Since energy is dissipated in the branch mode, the total energy decays at an effective damping rate ξ_{eff}. Figure 4 shows the E_0-dependence of this effective damping ξ_{eff}, in comparison with the analytical prediction of the previous section. As expected, the analytical approach corresponds to the limit of the numerical solution as E_0 tends to zero. As E_0 increases, the analytical approach increasingly overestimates the numerical effective damping. However, the ratio ξ_{eff}/E_0—constant in the analytical approach—remains finite: this constitutes the essential effect of branching on damping.

The influences of the design parameters ϕ_b, ξ_b, Ω and Γ on the effective damping, scaled by the initial energy,
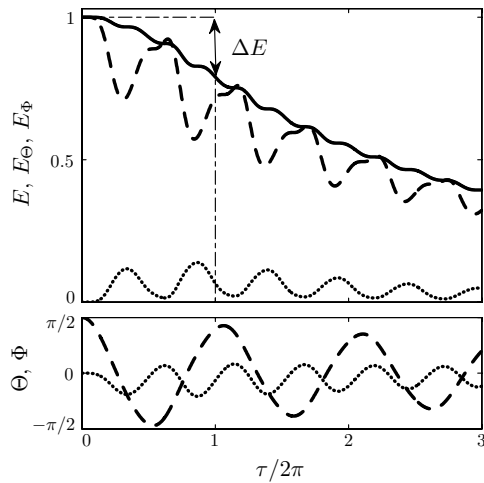
Figure 3. Typical evolution of the total energy, E (——), and modal energies, E_Θ (- - - -) and E_Φ ($\cdots\cdots$), with the respective evolution of the trunk angle, Θ (- - - -), and branch angle, Φ ($\cdots\cdots$), of the spring–mass model of a Y-shape, as a function of time over three periods of the trunk mode. The initial energy is $E_0 = 1$, i.e $\Theta_0 = \pi/2$, in the trunk mode only. The total energy decreases as a consequence of the energy nonlinearly transferred to the damped branch mode. The design parameters are set to $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$.
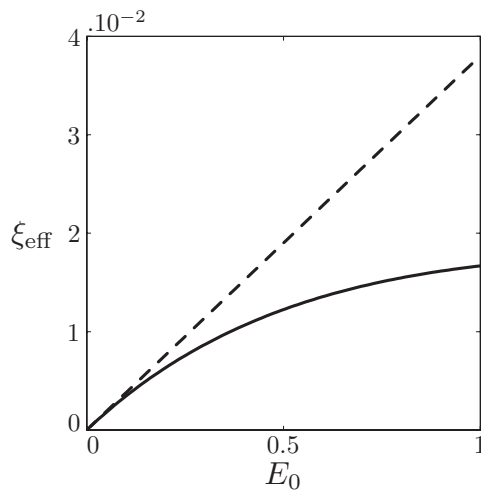

Figure 4. Effect of the initial energy level, E_0, on the effective damping, ξ_{eff}, of the spring–mass model of a Y-shape: (- - - -) analytical effective damping from the low energy approximation, (18); (——) numerical effective damping from the full dynamics integration of (7). The design parameters are set to the same values as in figure 3.

optimum effective damping at any energy, a good compromise would be to set the branching angle ϕ_b between $\pi/2$ and $2\pi/3$.

Not surprisingly, figure 5(*b*) shows an optimal branch mode damping ratio value at about 0.2. In fact, $\xi_b = 0$ obviously results in no effective damping since the energy cannot be dissipated anywhere in the undamped structure. In the other limit of high branch damping, the branch mode is critically damped so that the branches are locked with the trunk, resulting in a low nonlinear energy transfer and consequently a low effective damping mechanism. This classical behavior can be seen for other types of structure such as for taut cables (Krenk 2000). Figure 5(*b*) shows that significant effective damping is created by a large range of branch mode damping, ξ_b, as was also found in the low-energy analytical approach. Analogously, the typical shape of the Ω-dependence at low energy is also conserved when the energy increases, figure 5(*c*), with an optimal value of $\Omega \approx 1.8$ and a large range of frequency ratio leading to a significant effective damping $\xi_{eff} > 3\%$. Finally, the simple Γ-dependence on the effective damping is shown in figure 5(*d*). The modal mass ratio Γ has to be maximal in order to get the highest possible effective damping. For a high energy level, we observe that the effective damping ξ_{eff} is almost constant for Γ between 0.2 and 0.4.

3. Finite-element model of a Y-shape

The damping-by-branching mechanism described in the preceding section is now analyzed in the case of a more realistic continuous beam structure of a Y-shape. The same approach is used to demonstrate the effective damping: initial energy in the trunk, dissipation in the branches and effective damping evaluated by the total energy loss over one period of the trunk mode. Note that this model incorporates several differences from the previous one: a very large number of modes, symmetric and non-symmetric modes, non-localized mass and stiffness.

3.1. Model

The model consists of three assembled beams, figure 6(*a*). Each beam has a uniform circular cross-section and is made of a linearly elastic, isotropic and homogeneous material. The trunk, of length l_1 and diameter d_1, is clamped at the base. Two symmetrical branches, each of length l_2 and diameter d_2, are clamped at the tip of the trunk so that they each form an angle ϕ_b with the trunk direction. As in section 2, we analyze the free vibrations of the structure. To solve the equations of motion, numerical finite-element computations are performed using the CASTEM v.3M software (Verpeaux *et al* 1988). The finite-element model consists of Euler–Bernoulli beam elements, the trunk and each branch being described by ten mesh-elements. This refinement was found sufficient to describe the full dynamics of the system according to a convergence test. In order to take into account large amplitudes of motion, since we are interested in geometric nonlinearities, an incremental step-by-step procedure is used in
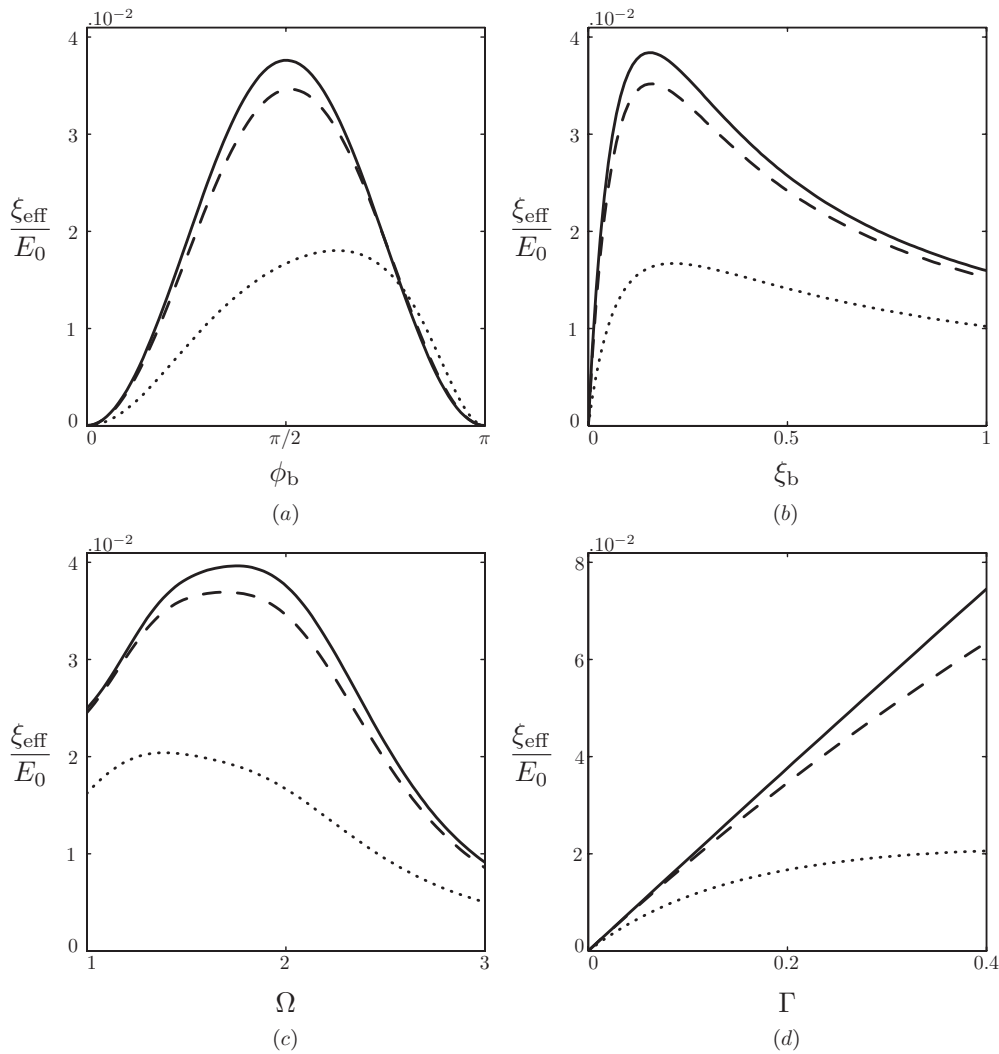
ξ_{eff}/E_0, are represented in figures 5(*a*)–(*d*) for three initial energy levels. As expected, the analytical and numerical approaches yield identical results for low energy levels and therefore are represented by the same curve for $E_0 = 0.01$. Consistently with figure 4, we observe that the analytical approach overestimates the numerical effective damping as E_0 increases. In figure 5(*a*), at low energy levels, the effective damping is proportional to $\sin^2 \phi_b$ as predicted by the analytical approach (18). The optimal branching angle ϕ_b shifts from $\pi/2$ to slightly higher values when the initial energy level increases. Therefore, in order to obtain an

Figure 5. Effects of the design parameters on the effective damping scaled by the initial energy, ξ_{eff}/E_0, of the spring–mass model of a Y-shape: $E_0 = 0.01$ (——); $E_0 = 0.1$ (- - - -); and $E_0 = 1$ ($\cdots\cdots$). Unless varied, the design parameter values are $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$. (*a*) Effect of the branching angle ϕ_b. (*b*) Effect of the branch mode damping ξ_b. (*c*) Effect of the branch/trunk modal frequency ratio Ω. (*d*) Effect of the branch/trunk modal mass ratio Γ.
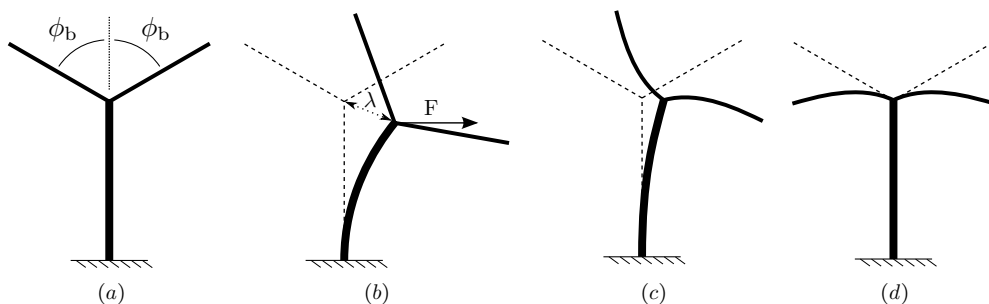

Figure 6. The continuous model of a Y-shape. (*a*) Geometry. (*b*) Static initial condition. (*c*) Trunk mode. (*d*) Damped branch mode.

the CASTEM v.3M software. This procedure uses an implicit Newmark scheme. All stiffnesses are updated at each step, including the elastic stiffness and the geometrical stiffness related to internal stress.

The first two modal shapes are given by modal analysis and are shown in figures 6(*c*) and (*d*). The branch mode,

figure 6(*d*), involves only bending of the branches as for the lumped parameter model of section 2, but the trunk mode, figure 6(*c*), involves mainly trunk bending, with a small amount of bending of the branches. Still, we will refer to this mode as the trunk mode for the sake of clarity and for comparison with the model of the previous section.

By analogy with (9), the initial condition is an initial deformation resulting from a horizontal static pull on the tip of the trunk, figure 6(b). The resulting initial energy E_0 is normalized so that it is equal to 1 when the deflection of the trunk is equal to its length such that $\lambda = l_1$. This initial condition corresponds to a distribution of the total deformation energy as follows: 94.3% in the trunk mode, 0% in the branch mode and 5.7% in all other modes. By analogy with section 2, two dimensionless parameters are chosen: the frequency ratio $\Omega = \omega_2/\omega_1$ and a mass ratio $\Gamma = (l_1 m_2)/(l_2 m_1)$, where ω_1, ω_2, and m_1, m_2 are the modal angular frequencies and the modal masses of the trunk mode and branch mode, respectively. Energy dissipation is introduced artificially on the branch mode only. To do so, a damping matrix $[C]$ is derived from the mass matrix $[M]$ of the finite-element model, from the branch mode modal shape vector denoted $\underline{\psi_b}$ and with the aimed branch mode damping ratio denoted $\overline{\xi_b}$ as follows:

$$[C] = \frac{2\xi_b \omega_2}{m_2}([M]\underline{\psi_b}) \otimes ([M]\underline{\psi_b}), \qquad (20)$$

where $\otimes$ denotes the tensor product. Note that this form of the damping matrix is not related to a particular physical choice of Rayleigh damping but is built *ad hoc* to evidence the specific role of branch mode damping in the model energy transfer.

The resulting effective damping mechanism is studied with the same definition of the effective damping rate, ξ_{eff}, as in section 2.2, and for the same reference values of the design parameters $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$.

3.2. Results

The simulated dynamics of the continuous Y-shape yields a similar time evolution of the total energy to that of the lumped parameter model, figure 3. The corresponding effective damping rate is plotted in figure 7 as a function of the normalized initial energy E_0. As in figure 4, the effective damping rate increases with the initial energy level, quasi-linearly at first, reaching several percent for high levels of initial energy. This is a first indication of the robustness of the effect of branching on damping in a more realistic structure.

As in section 2, the effects of the design parameters ϕ_b, ξ_b, Ω and Γ on the effective damping scaled by the initial energy, ξ_{eff}/E_0, are represented in figures 8(a)–(d) for three initial energy levels. Some differences appear for the continuous model as expected. First, the branching effect on the effective damping is clearly maximal for larger branching angles, $\phi_b \approx 2\pi/3$, rather than $\pi/2$, figure 8(a). Second, the effective damping stabilizes or even slightly increases with the branch mode damping ξ_b, figure 8(b), instead of decreasing after $\xi_b \approx 0.2$. Third, the effective damping is higher at a modal frequency ratio $\Omega = 3$ than $\Omega = 2$, figure 8(c), suggesting a richer pattern of internal resonances. Singularly, we observe that the effective damping vanishes here for $\Omega = 1$. This is explained by the fact that, as Γ is kept constant, $\Omega = 1$ represents a physical limit where the length of the branches compared to the trunk tends toward 0. Finally, the effective
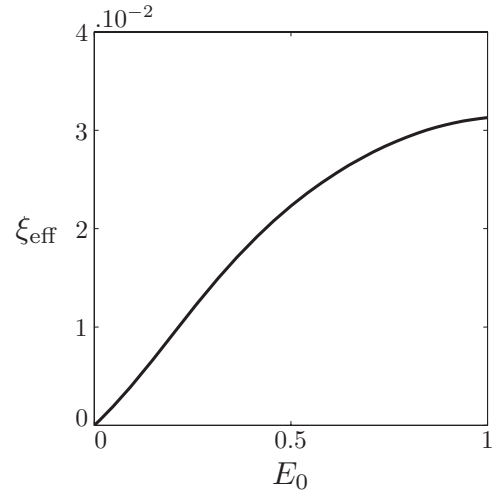

Figure 7. Effect of the initial energy level, E_0, on the effective damping, ξ_{eff}, of the continuous model of a Y-shape. The design parameters are set to $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$.

damping increases with the modal mass ratio Γ but in a more complex way, figure 8(d).

To further characterize the dynamics of the system, we consider now its response to a harmonic loading. From the rest position, the structure is forced with an oscillating torque of frequency Ω_f and amplitude M_0 near the base of the trunk at one-tenth of the height of the trunk. The steady state oscillation amplitude, λ, is shown in figure 9, relative to the static response λ_{static}. Because of the damping-by-branching nonlinear mechanism, the resonance peaks at $\Omega_f = 1$ and $\Omega_f = 2$ are damped though there is no damping in the trunk mode. One can also estimate an effective damping ratio ξ_{eff}, considering that the amplitude at the resonance peak divided by the static response is proportional to $1/(2\xi_{eff})$. One obtains $\xi_{eff} \approx 5\%$ for the first peak resonance, where the level of energy E associated with the amplitude λ is about 0.9. This value of effective damping is consistent with the case of the previous pull-and-release loading in this range of energy, see figure 7.

In conclusion, it appears that the main features of the damping-by-branching mechanism are still present in this more realistic Y-shaped structure.

4. Two bioinspired branched structures

Based on the results of sections 2 and 3, two bioinspired branched structures are considered, figure 10. These two bioinspired structures have the same trunk of length l_1 and diameter d_1 as the model of section 3, so that the initial condition is the same, with an initial bending energy in the trunk only, as in figure 6(b), and with the same definition of the initial energy E_0.

The first bioinspired structure, shown in figure 10(a), is a two-generation T-shaped structure designed so that $\phi_b = \pi/2$ at each branching. The ratios of branch length and diameter are respectively the same between orders of branching, i.e. $l_2/l_1 = l_3/l_2$ and $d_2/d_1 = d_3/d_2$. They are chosen so that the
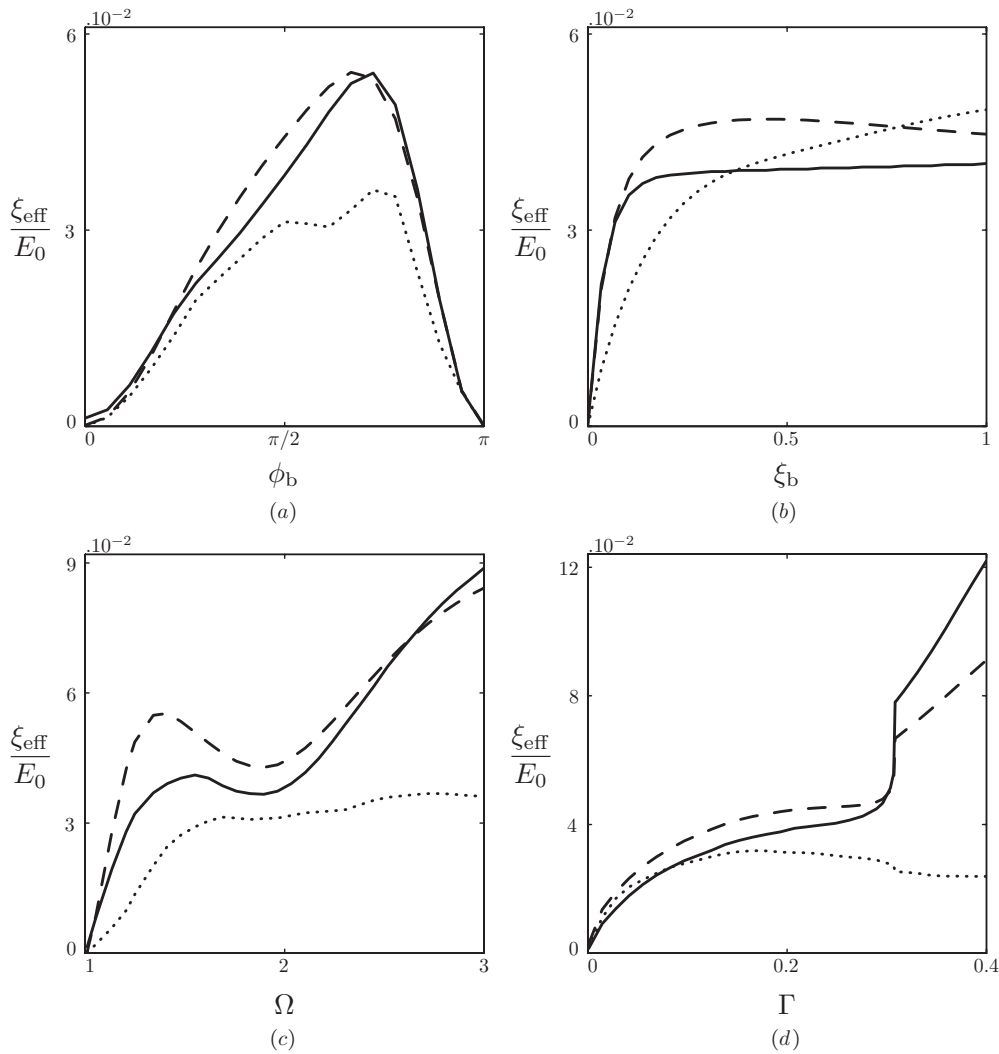
Figure 8. Effects of the design parameters on the effective damping scaled by the initial energy, ξ_{eff}/E_0, of the continuous model of a Y-shape: $E_0 = 0.01$ (——); $E_0 = 0.1$ (- - - -); and $E_0 = 1$ ($\cdots\cdots$). Unless varied, the design parameter values are $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$. (*a*) Effect of the branching angle ϕ_b. (*b*) Effect of the branch mode damping ξ_b. (*c*) Effect of the branch/trunk modal frequencies ratio Ω. (*d*) Effect of the branch/trunk modal mass ratio Γ.

modal frequency ratio between the trunk mode, figure 10(*b*), and the last-order branch mode, figure 10(*c*), is 1:2 and so that the modal mass ratio is 0.2. With the same procedure as in the previous section, (20), a damping rate of 0.2 is introduced in this last-order branch mode only.

The second bioinspired structure, shown in figure 10(*d*), consists of a double Y-shaped pattern with an added level of branching at 3/4 of the height of the trunk. Both levels of branching have a branching angle $\phi_b = 2\pi/3$ and are designed so that the modal frequency ratio between the trunk mode, figure 10(*e*), and the large branch mode, figure 10(*f*), is 2, and the modal frequency ratio between the trunk mode and the small branch mode, figure 10(*g*), is 3. A damping of 0.2 is introduced in the two branch modes only. The resulting effective damping is studied with the same damping criterion as in section 2.2 and is plotted in figure 11 as a function of the normalized initial energy. In both structures, the effective damping reaches several per cent ($\approx 3\%$) for high levels of initial energy, roughly corresponding to a third of the initial energy being dissipated after one period of the first mode.

These results on two different bioinspired branched structures show that the damping-by-branching mechanism seems to be robust regarding the branching scheme.

5. Discussion and conclusion

At this stage, one may consider the results of the preceding sections in relation to the proposed aim of the paper: to identify and characterize the elementary mechanism that causes nonlinear modal energy transfer and amplitude-dependent damping in branched structures. In section 2, we have shown that branching is the key ingredient needed to obtain the modal energy transfer and the resulting effective damping that several authors had suspected. Sections 3 and 4 confirm that the essential features of this damping-by-branching are present even in more complicated branched models.

Clearly, the mechanism found here is complementary to the tuned-mass damper mechanism described by Spatz *et al* (2007) in trees. The present damping-by-branching
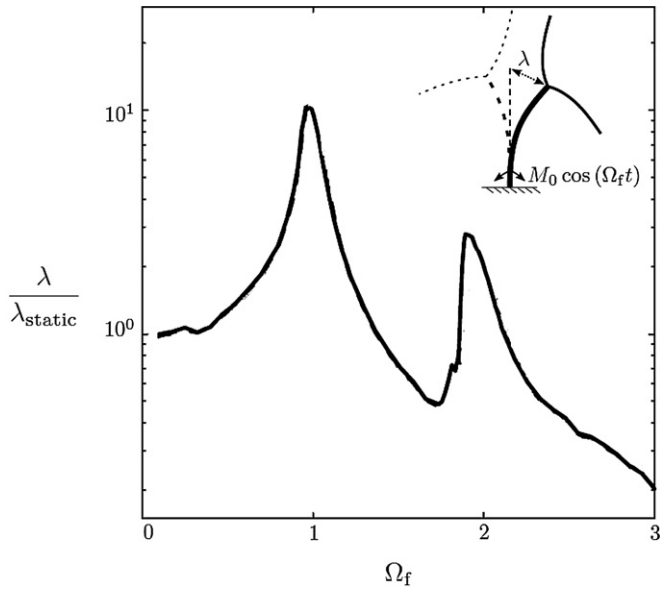
Figure 9. Scaled steady-state displacement of the continuous model of a Y-shape in response to an oscillating torque of frequency Ω_f near the base of the trunk. Reference values of the design parameters are used, $\phi_b = \pi/2$, $\xi_b = 0.2$, $\Omega = 2$ and $\Gamma = 0.2$.

Figure 11. Effect of the initial energy level, E_0, on the effective damping, ξ_{eff}, of the two bioinspired structures. ($\bullet$) Ramified T-shaped structure; ($\blacklozenge$) double-branched Y-shaped structure.

mechanism consists of two essential characteristics: (i) it is not associated with the condition of identical modal frequencies of the trunk mode and of the branch mode; (ii) since it originates in geometrical effects, the larger the amplitude of motion, the higher the effective damping. As this mechanism is specific to damp out large-amplitude motions, it can be useful only in very slender and flexible structures where the limit elastic stress is reached only during extreme dynamical events. In this type of structure, we have shown that this damping-by-branching can be achieved with some requirements on the design parameters: a m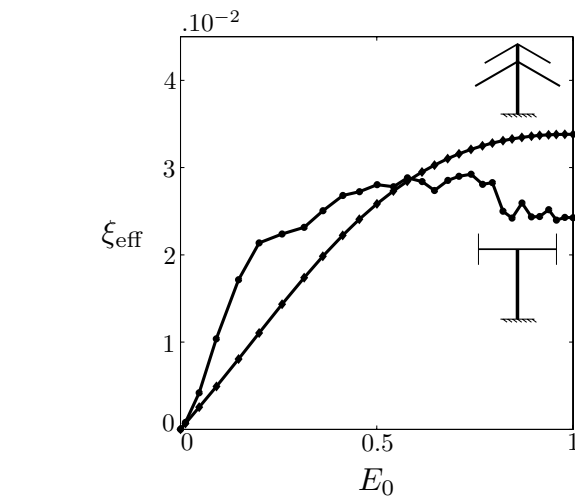odal frequency ratio between the trunk mode and the branch mode near 1:2, a branch damping ratio about 0.2, and the highest possible modal mass ratio. All these results suggest that modal energy transfer and the resulting damping-by-branching are robust effects at large amplitudes of motion.

Before generalizing our results, discussion is needed of some of the assumptions made to derive them. Firstly, the analyses pertaining to the effective damping have been made on the dynamical responses to pull-and-release initial loading or to a harmonic excitation of the trunk. This choice was made so that only the nonlinear geometrical effects could cause the effective damping of the structure. If other classical types of loading were considered such as an initial impulse, or a random forcing (Humar 2002), energy would have been given to all the modes of the branched structure. Although this would make the global energy balance more complex to analyze, the
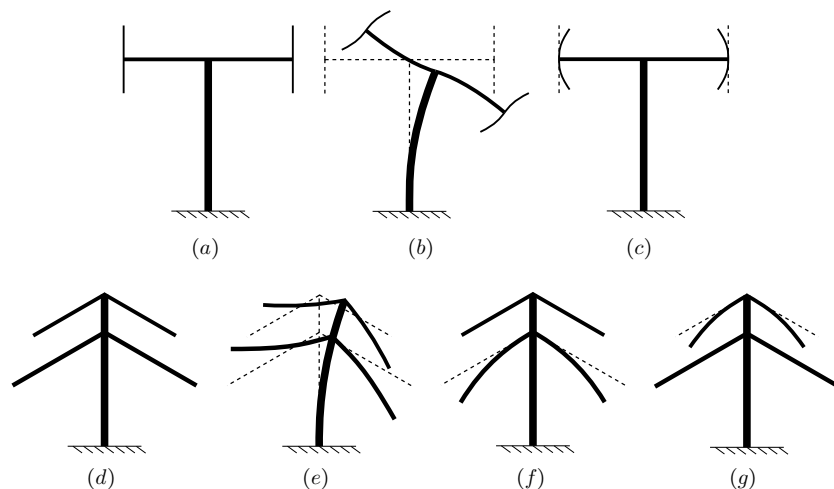

Figure 10. Two bioinspired branched structures. (*a*) A two-order ramified T-shaped structure, and its modes of interest: (*b*) trunk mode, (*c*) damped branch mode. (*d*) A double-branched Y-shaped structure, and its modes of interest: (*e*) trunk mode, (*f*) damped large branch mode, (*g*) damped small branch mode.

nonlinear geometric terms responsible for the modal energy transfer (7) would still be present but the resulting effective damping would not be simply quantified. Secondly, we have always considered perfectly symmetric and plane structures. If asymmetry between the branches is introduced in the model of section 2, a linear coupling is introduced between the trunk and branch angles of motion so that the energy balance analysis becomes more complex. By some aspects, such a change is expected to bring similar effects as when introducing higher modes, as was done in sections 3 and 4: the mechanism is qualitatively the same. Similarly, if three-dimensional effects are introduced, such as torsion or multiple 3D branching as in real trees, a much larger number of degrees of freedom is needed in order to describe the dynamics of the structure. However, the results of sections 3 and 4 show that complicating the modal content of the model does not impact the existence of the modal transfer mechanism and the resulting effective damping. Moreover, Rodriguez *et al* (2008) showed that there exist no significant differences between the dynamics of an actual tree architecture and that of an idealized one.

Finally, an important requirement of this damping-by-branching mechanism is the damping of the branch mode. We have shown that the optimal damping ratio for this mode is approximately $\xi_b = 20\%$. Under this condition, for a general structure of mass m, stiffness k, the physical damping denoted c scales as $\xi_b\sqrt{mk}$. In other words, such a damping ratio cannot be expected for heavy structures. However, for light structures, physical phenomena such as drag-induced damping for a slender structure vibrating in a cross flow often reach this order of magnitude for the damping ratio, see Blevins (1990). More generally, the question of how branched systems move in a fluid environment is important in practice. In fact, our analysis on damping originated in the dynamics of trees under wind-loading (Spatz *et al* 2007). In the interaction between a branched structure and flow, several distinct effects may be expected (Blevins 1990, Païdoussis *et al* 2011). Firstly, even in the absence of flow, just the presence of a fluid around the structure causes damping. This damping is present in all modes, is amplitude dependent, and introduces nonlinear coupling between modes. Flow-induced damping may also appear in addition. These effects may be gathered under the generic term of aeroelastic or hydroelastic damping. Secondly, added mass and added stiffness effects appear and may alter the essential dynamical characteristics of the branched structure, such as frequencies and modal shapes. These effects are more pronounced in water. Finally, flow may cause a large variety of loadings through mechanisms such as turbulence excitation or wake interactions. From this list, it appears that all the key parameters involved in the mechanism of damping-by-branching are affected by a fluid environment: modal damping, frequencies, modal shapes and external excitations. In our models, only artificial loadings and modal damping have been investigated so that a systematic analysis of these effects is clearly needed on the basis of the simple framework presented in this paper.

In the design of a slender, flexible and light structure that may encounter extreme dynamical loadings, a simple rule to follow is to introduce branching and a significant damping ratio between branches. Secondary rules, aiming at optimizing the efficiency of this damping mechanism, are to set the ratio between the modal frequency of the branch mode and the trunk mode near 1:2 and the modal mass ratio as high as possible. Although a design rule based on the ratio of modal frequencies and masses is not common, it should be noted that the requirements are not strict, as we have shown in sections 2 and 3 that damping-by-branching is robust and is significant for a wide range of modal frequency and mass ratios. The 1:2 rule is only indicative, as it is related to the original internal resonance condition between the branch and trunk modes. At this point, the concept of damping-by-branching has only been demonstrated to exist theoretically and numerically. Although it has been inspired by the observation of natural systems, it evidently needs to be explored experimentally on man-made structures.

Acknowledgments

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