



## Review

## Oscillation damping in trees

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## ABSTRACT

Oscillation damping is of vital importance for trees to withstand strong gusty winds. Tree adaptation to wind loading takes place over a long time and during a storm only passive damping mechanisms can reduce the impact of the wind on trunk and roots. Structural damping, a phenomenon, which is associated with the conspicuous movements of the branches relative to the trunk is of particular importance. Primary and higher order branches can be seen as multiple tuned mass dampers. Moreover, as the frequency bands overlap within branches and between primary branches and the entire tree, resonance energy transfer can distribute mechanical energy over the entire tree, such that it is dissipated more effectively than in a tree with stiff branches and not so much focused on the tree trunk and the roots.

Theoretical studies using modal analysis and finite element methods have supported these assertions. Next to “multiple mass damping” and “multiple resonance damping”, both characterized by linear coupling between the elements, a third non linear mode, operative at large amplitudes has been identified: “damping by branching”. In all these not mutually exclusive concepts frequency tuning between the elements appears to be a fundamental requisite.

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## Contents

1. Introduction .....	66
2. Technical applications of oscillation damping .....	67
3. Experimental work .....	67
3.1. Pull and release experiments .....	67
3.2. Observation of tree sway .....	67
4. Theoretical considerations .....	68
4.1. Mass damping and finite element analyses .....	68
4.2. Multiple resonance damping .....	68
4.3. Modal analyses .....	69
4.4. Damping by branching .....	69
5. Suggestions for future research .....	69
6. Conclusions .....	70
Acknowledgements .....	70
References .....	70

## 1. Introduction

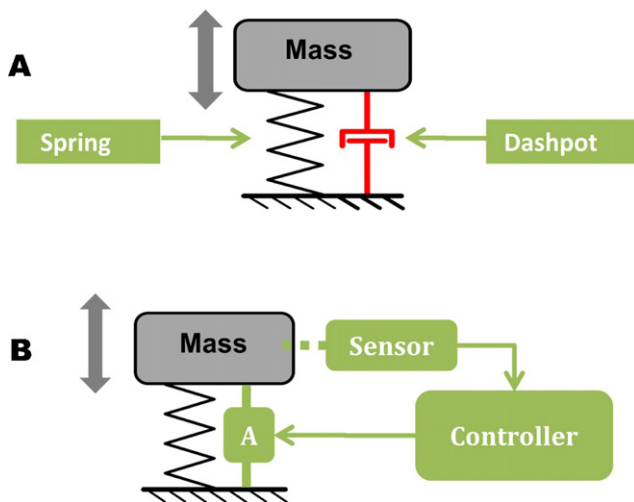
“Life in Moving Fluids” [1] provides a comprehensive description of the biological implications of hydrodynamics. Wind forces are the most critical forces that a land plant has to withstand [2,3].

The mechanical stability of trees under static wind loads has been reviewed [4]. It is particular important that trees and other plants can react to wind in a flexible manner. Stem and branches bend towards the lee side. This “streamlining” [5–8] leads to a significant reduction of the sailing area and to some extent also to the drag coefficient. Leaves also reconfigure alignment in the wind, which further reduces drag [9,10].

Static wind can only be realized in wind tunnels. In reality wind is always dynamic with a broad range of frequencies [11–13]. Trees and other plants are, therefore, likely to be excited to sway

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**Fig. 1.** Schematic representation of different principles for oscillation damping. (A) passive damping by dissipation of energy in a dashpot (compare the Voigt/Kelvin model for viscoelastic materials [20]). (B) active counteraction requiring a sensor, a controller, and an actuator A.

[11,14–19]. Unless these oscillations would be damped, a “resonance catastrophe” could lead to stem breakage or uprooting.

If friction among different plants or among different side organs [16,19] and friction in the root soil system are set aside, there are two principle sources of oscillation damping. Mechanical energy imposed by gusty winds can be converted to heat by viscous damping in the material, or it can be dissipated to the surrounding fluid, i.e., aerodynamic damping [20]. Branches can often be seen not to sway in line and in phase with the trunk of a tree, particularly in strong gusty winds. As discussed in sections 3.1 and 3.2, experiments show that this leads to an amplification of viscous damping as well as of aerodynamic damping, referred to as structural damping [21]. Three, not mutually exclusive, concepts have been proposed to describe this phenomenon: tuned mass damping [22,23], resonance energy transfer [24], damping by branching [25]. These concepts will be delineated in detail in Sections 3.2, 4.2 and 4.4.

## 2. Technical applications of oscillation damping

Under conditions of dynamic mechanical loads, which may result from gusty winds, water waves, or even from earthquakes, oscillation damping is of vital importance for the stability of trees or man-made structures. It is also an important aspect of posture control and control of movements [26]. There are several mechanisms by which unwanted or even potentially dangerous oscillations can be minimized: dissipation of mechanical energy, passive counteraction combined with energy dissipation, active counteraction.

These principles can be illustrated by the way they are applied in architecture and mechanical engineering (Fig. 1). Dissipation of energy is usually accomplished by a combination of springs and shock absorbers often referred to as dashpots. A car with malfunctioning shock absorbers will be dangerously unstable on bumpy roads. A well-known example of the application of passive counteraction is the Taipei tower at Taipei, Taiwan. As a tuned mass damper a 660 ton spherical body is suspended on steel cables and equipped with hydraulic dampers, such that it acts as a damped pendulum which swings with the same frequency as the entire building but with a 180° phase shift. This way it counteracts swaying of the building induced by the frequent earthquakes and yearly typhoons in this region. Other examples of passive counteractions are tuned mass dampers on tall chimneys or on wide span

constructions such as the London Millenium Bridge or electric power lines.

Anti rolling tanks in ships were developed as early as 1889. The ship is equipped with two water tanks above the water line, one on each side. The tanks are connected by pipes, such that water can flow back and forth from one side to the other. This counteracts rolling movements of the ship. Tuning the filling of the tanks and the size of the pipes to the eigenfrequency of the ship, a 180° phase shift is attained, while dissipation of energy occurs via the flow of water through the pipes.

Alternative designs to reduce rolling in ships are automatically operated fin stabilizers localized below the water line. Such an active counteracting device requires sensors and actuators steered by a feed back controller (Fig. 1), or more advanced feed forward computing system. An active anti-roll stabilizer is also incorporated in some technically high quality cars.

## 3. Experimental work

### 3.1. Pull and release experiments

Earlier observations of conifer tree sway and oscillation damping after pull and release tests, have been reviewed [27]. The pull and release experiments on 26 years old Sitka spruce trees are particularly noteworthy: sway studies predicted that there is more aerodynamic drag than expected from wind drag studies, even when the influence of interference of branches with those of neighboring trees and viscous damping in the stem was subtracted [16]. A linear dependence of the damping ratio on the relative velocity was found in Norway spruce trees that was attributed it to viscous damping alone [28]. However, it should be considered that aerodynamic damping follows an exponential law with an exponent somewhere between 1 and 2 due to streamlining [5–8], such that it is difficult to separate the contributions of viscous and aerodynamic damping from a sway decay curve alone.

In pull and release tests on a young Douglas-fir tree viscous damping in the debranched stem amounted to 13% and aerodynamic damping for maximally 20%, (calculated on the basis of photographs in the direction of sway of all branches, but for a supposed rigid structure), such that structural damping accounted for two thirds of the overall damping observed for the intact tree [24]. Apparently the flexibility of the branches allows for an effective oscillation damping. Pull and release experiments were also carried out on deciduous trees [29]. In experiments on Bradford pears, leaves were shown to contribute significantly to oscillation damping [30]. Prior to these studies structural damping [21] was observed in pull and release experiments on *Arundo donax* [31,32].

### 3.2. Observation of tree sway

An alternative experimental approach is to record tree motion with a set of sensors distributed over the tree trunk and the branches in conjunction with monitoring wind speed and its direction in the immediate vicinity [11,13,15,33]. Natural sway frequencies and damping ratios of intact, partially debranched, and fully debranched Douglas-fir trees were measured [34]. The results suggested that branches should not be viewed as a series of masses lumped to the stem, but rather as individual damped harmonic oscillators coupled to the stem (compare [18]). This was confirmed after observing a high degree of damping during natural sway of conifers, two eucalypts and a palm tree under conditions of strong winds [23]. The high damping efficiency was explained to result from the dynamic sway of the branches with respect to the trunk referring to the concept of multiple mass damping (Fig. 2).

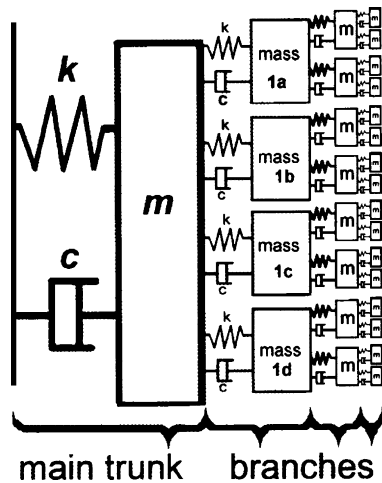


Fig. 2. A model for multiple mass damping, adopted from [23]. Springs are denoted by  $k$ , dampers by  $c$  and masses by  $m$ . Masses 1a–1d and others as well as the corresponding springs and dampers may differ from each other.

#### 4. Theoretical considerations

##### 4.1. Mass damping and finite element analyses

Mass damping devices, which combine passive counteraction and energy dissipation have widely been applied in architecture and mechanical engineering (see section 2). That the principle applies also to trees has to our knowledge first been described in an unpublished PhD thesis [35] as referred to in [11]: “In this way energy absorbed by the whole tree can effectively be transferred through the branches and sub-branches to the needles and re-emitted as wake turbulence at much higher frequencies”. More recently a similar dynamic model of trees was advanced [22] in which the branches are seen as cantilevers performing complex sway motions. This was suggested before from comparisons of a mathematical model to describe the dynamic response of a spruce tree and its measured behavior [18]. In pull and release experiments on young maritime pines branches were found to oscillate with the same frequency as the entire tree, but with a certain phase shift [36]. These studies were accompanied by finite element calculations [37], which suggest that the flexibility of the branches determines the degree to which structural damping comes into play: calculated damping ratios are increased by a factor of 2 as compared to a model where the branches are assumed to be rigid. Tree sway under natural conditions [34] has also been simulated using finite element analyses [38]. The modulus of elasticity in the longitudinal direction of the branches appeared to be a critical parameter in structural damping.

##### 4.2. Multiple resonance damping

Further refinements of the concept of structural damping were based on the experiments [24] described in section 3.1. The natural frequency of every branch could be calculated on the basis of an analytical approach developed earlier [39]. The calculations used data from measurements of length, basal diameter, taper, and mass distribution for every branch, as well as the modulus of elasticity and its variation along the length of the branch [24]. The natural frequencies of all larger branches are in the same range or only slightly larger than the natural frequency of the entire tree (Fig. 3). A similar relation between the natural frequency of a tree and that of its branches has been reported for *Pinus pinaster* [40], *Platanus × hispanica* [41] and *Tilia cordata* (Fig. 4). Resonance energy transfer (a term reminiscent of phenomena in the molecular

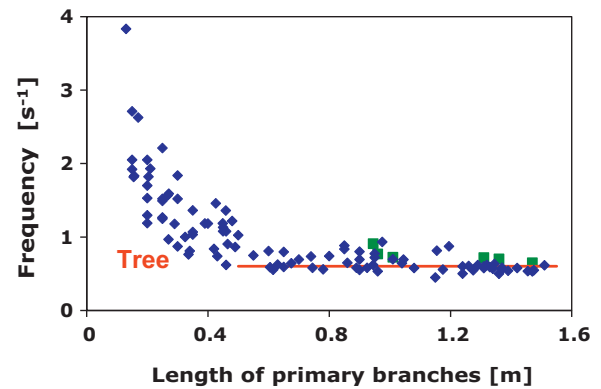


Fig. 3. The distribution of natural frequencies of a Douglas-fir tree (solid line) and its branches as measured directly (squares) and calculated according to [39] (diamonds). The data entering the calculations, mass, basal diameter and taper, length of each branch, the modulus of elasticity and its change along the length of the branch are described in detail in [24]. The results of the calculations are plotted as function of the length of the branches.

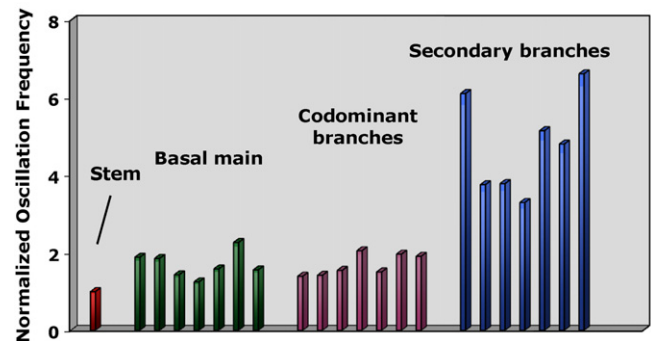


Fig. 4. The natural frequencies of a *Tilia cordata* tree and its branches [41] as estimated on the basis of the length, diameters and the taper of the plant organs, and a modulus of elasticity for green wood as listed in [20]. The approximations show that the natural frequencies of the entire tree, the main branches and the secondary branches are not far from each other.

domain [42]), is possible if the natural frequencies in a system of coupled damped oscillators are close enough (Fig. 5). “This transfer of energy between tree elements requires the frequency bands of each element to overlap, such that the range of natural frequencies for secondary order branches overlaps for primary branches which,

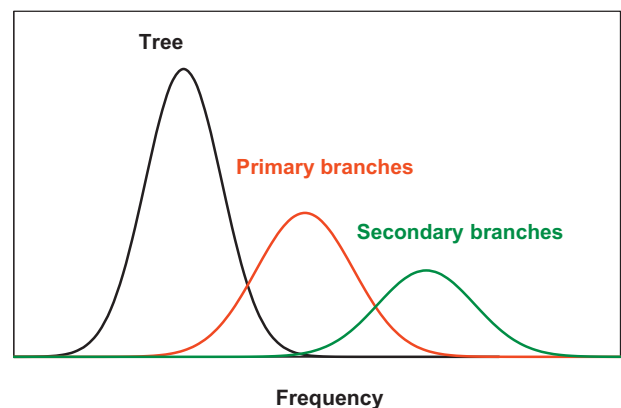


Fig. 5. Resonance spectra for a tree, a first order branch, and a second order branch, typical for damped oscillators, are represented schematically. If the spectra overlap, energy can be transferred, either by direct mechanical coupling or via the surrounding fluid, from second order branches to first order branches and from first order branches to the trunk, and vice versa. This way the mechanical energy can be distributed over the entire tree.

in turn, overlaps that for the whole tree” [38]. If there is no large gap in the distribution of natural frequencies, energy can be distributed over the entire tree. Thus energy, stored as strain energy in the stem and the roots before release, can be transferred to other branches and/or higher order branches after release and will be dissipated most effectively in the periphery of the tree both by viscous and by aerodynamic damping.

This concept of “multiple resonance damping” [24] or “resonance energy transfer” is also important for tree motion under natural conditions. Tree sway is induced by wind forces acting primarily on branches perpendicular to the direction of the wind. Resonance will transfer the energy to the other branches of the tree, such that it not so much focused onto the trunk and the roots as compared to a structure with rigid side organs. This way the tree in gusty winds can bear up against peak wind speeds, which would lead to uprooting or stem breakage under exposure to steady winds of the same speeds.

#### 4.3. Modal analyses

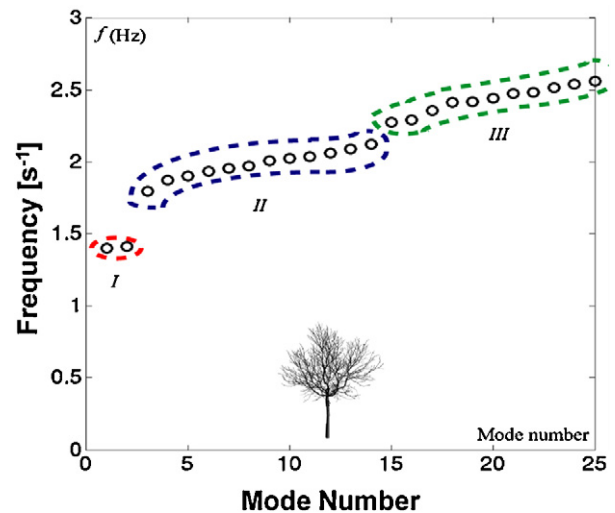
Dynamics and damping of a complex structure like a tree can also be assessed using standard mechanical tools, particularly using modal analysis. Eigenmodes consist of particular free motions of the whole structure where all parts move at the same frequency and phase. Each eigenmode is characterized by its modal frequency, modal mass, and spatial deformation termed “modal shape”. The motion at any point of a tree can then be expressed as a sum of each modal contribution. Eigenmodes also have their own damping rate resulting from the different damping processes such as aerodynamic effects and viscous damping. Since wind loads excite tree sway only at a particular range of frequencies, it is possible to select a finite number of relevant eigenmodes, resulting in a simpler dynamical tree model with fewer degrees of freedom. This method has been used for simple and realistic animated tree motions with low computation cost [43].

Determining the modal shapes of a tree and their corresponding frequencies can be done both experimentally and theoretically by using finite-element models, as was done for a walnut tree [44]. An accurate description of the tree topology and mechanical characteristics is required for numerical investigations of the models used [37,38,40,43–45].

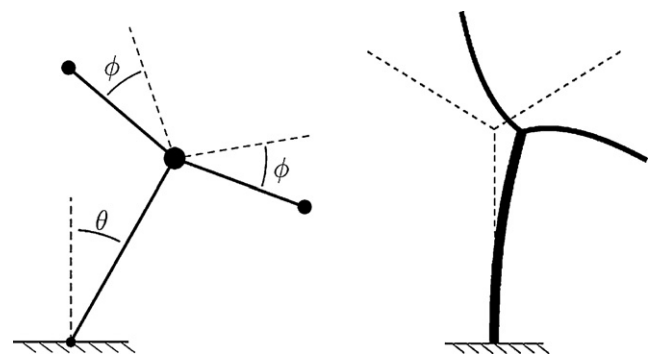
Despite the complexity of individual tree topology, scaling laws could be derived from an idealized branched model using few allometric parameters related to actual tree geometries [44]. This simplified model “explained most of the spatiotemporal characteristics of modes that involved the trunk and branches, especially for sympodial trees” [44]. Two particular features of the modal dynamics of trees are that modal frequencies are close to each other (Fig. 6) and corresponding modal shapes are spatially defined within the tree architecture: the first set of modes involves mainly trunk bending, the second set involves the first order branches, leaving the trunk almost unstrained, the third set of modes involves second order branches, leaving parent segments almost unstrained, and so on.

#### 4.4. Damping by branching

Only recently it was realized that a high modal density is classically favorable for non-linear energy exchange [25]. Indeed a two degree-of-freedom simplified tree model (Fig. 7) is suitable to demonstrate, that large amplitude motions lead to non-linear modal energy exchanges. These transfers are non-linear in the sense that they appear only for large amplitude motions. They are due to centrifugal excitation of a mode involving branch bending induced from trunk swaying. Once this energy has been non-linearly transferred to the branch mode, it can be dissipated



**Fig. 6.** Modal frequencies calculated for a model walnut tree, adopted from [44]. Frequency range I refers to trunk modes, range II refers to first order branch modes, and range III to second order branch modes. The modal frequencies are close to each other, a prerequisite for energy transfer between different modes.



**Fig. 7.** The two degree-of-freedom model and the continuous Y-shaped model in which damping by branching can be demonstrated, adopted from [25]. The angle  $\theta$  represents the trunk motion and the angle  $\phi$  the symmetrical motion of the branches. Other than in linear damping mechanisms, the efficiency of damping increases with the initial energy of excitation. Non linear damping occurs over a broad range of branch/trunk modal frequency ratios with a maximum around 2. If asymmetry between the branches is introduced in the model, an additional linear coupling between the trunk angle and the branch angles of motion complicate the analysis but the geometrical nonlinear terms responsible for the energy transfer would still be present.

resulting in an effective damping of the trunk. This “damping by branching” mechanism is characterized as being closely linked to the branching topology, and is based on a non-linear resonance most effective at a 1:2 ratio between the trunk mode frequency and the branch mode frequency [25].

There is yet no experimental proof of such mechanism in trees. Still, modal non-linear coupling associated with such an arrangement of modal shapes in a tree would be consistent with a non-linearly amplitude-dependent protective mechanism of the trunk and the roots through energy transfer toward higher modes in the branches.

#### 5. Suggestions for future research

Usually new experimental techniques pave the way to new insights and to a deeper understanding of natural phenomena. Particularly promising seems to be the Tree Motion Sensor (argus electronics GmbH, Rostock, Germany), available since 2012. If a multitude of these devices are attached to the trunk as well as first



and higher order branches of a tree, the movement of the branches relative to the trunk can be accurately recorded. Such records combined with FEM calculations for the same tree, will potentially give answers to a number of questions: What are the differences that the architecture of gymnosperms and angiosperms makes to their relative damping efficiency? Will a tree growing in the open with no constraints on light availability but with the highest wind loading have the most efficient structural damping and will trees in a canopy which need to grow branches upwards and into gaps to obtain light have less efficient damping? Will trees, having been de-branched to reduce their sailing area, be more endangered in gusty winds due to less efficient structural damping?

Related is the question whether under a biomechanical point of view optimization of trees is not only a compromise between light capture and stability against mechanical loads, gravitational forces and wind loads averaged over a long time period, and is effective structural damping just a trade off? Or is the flexibility of the branches and with it the efficiency of structural damping also optimized during the development under the influence of exposure to turbulent wind? Partial answers can probably be obtained by studying trees grown under very different wind exposure, another by studying mutant poplar trees.

Laboratory studies could provide information on how the efficiency of viscous damping in green wood is related to wood density and wood structure, in particular in tension wood and compression wood. Does flexure wood [46] allow for more efficient viscous damping and structural damping? Is the efficiency of viscous damping influenced by repeated flexing?

Pull and release experiments or observation of tree sway of deciduous trees with or without leaves could lead to a better assessment of aerodynamic damping. Since structural damping is an enhancement of viscous damping and aerodynamic damping as compared to a structure with stiff side organs, what are the relative efficiencies of the different modes of damping in different trees? Can such findings be generalized?

Modal analysis and FEM analyses on complex structures, preferentially structures resembling trees, could answer the question at which amplitudes of motion non-linear damping by branching comes into play in trees.

## 6. Conclusions

In high gusty winds branches can be seen to sway out of phase and not in line with trunk sway, often with much larger amplitudes than those of the trunk. Experimental and theoretical studies lead to the conclusion that movements of the branches relative to the trunk contribute significantly to oscillation damping, a phenomenon called structural damping. A closer analysis reveals that primary and higher order branches act as tuned mass dampers where viscous damping in the wood and energy dissipation to the surrounding medium is greatly enhanced in comparison with a tree with stiff branches.

Calculations of the natural frequencies of the major primary branches in several trees have shown that they are close to the natural frequency of the entire tree [24,40,41]. Moreover theoretical analyses have revealed a continuum of modal frequencies towards higher order branch modes in complex structures resembling trees [44]. If the frequency bands overlap sufficiently, there can be resonance energy transfer within branches and between branches and trunk, either by direct mechanical coupling or via the surrounding fluid. Therefore, the mechanical energy imposed by gusty winds can be distributed over the entire tree and not so much focused on the trunk and the roots—the vital parts of the tree.

Indeed, frequency tuning appears to be a fundamental requisite for distribution and effective dissipation of the mechanical energy,

be it by linear or non-linear coupling. The three not mutually exclusive concepts described here require frequencies between trunk and branches to lie optimally at a ratio of 1 (for the tuned mass damping and multiple resonance damping mechanisms) and a ratio of 2 (for the damping by branching mechanism). Further experiments, particular on deciduous trees, are desirable for additional support of these concepts.

### Box 1: Modes of oscillation damping

1. Viscous damping: In oscillations where the amplitudes of motion exceed the linear elastic range of the material (here green wood) mechanical energy is converted to heat, leading to gradually decreasing amplitudes of an oscillation.
2. Aerodynamic damping: Movement of objects (here trees) in a viscous fluid (here air) leads to dissipation of energy to the surrounding fluid and thus to gradually decreasing amplitudes of an oscillation.
3. Structural damping: Large movements of branches relative to the trunk of a tree can lead to an increase of viscous damping and aerodynamic damping (particularly in the periphery of the tree) as compared to a tree with stiff branches firmly attached to the trunk, such that trunk and branches move in line and in phase together.
4. Damping by interaction with neighboring trees: mechanical energy is either transferred to trees in contact or converted to heat by friction between the different side organs.
5. Damping by root soil interaction: Movements of the trunk will lead to strains in the roots and unless a tree is firmly embedded even movements of the roots in the soil. This leads to shear between roots and soil and within the soil and thus to conversion of mechanical energy to heat.

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