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CREEPING AXISYMMETRIC MHD FLOW ABOUT A SPHERE TRANSLATING PARALLEL WITH A UNIFORM AMBIENT MAGNETIC FIELD

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This work investigates the slow viscous MHD axisymmetric flow about a solid sphere with a radius a translating parallel to a uniform magnetic field with a magnitude $B > 0$ in a quiescent conducting Newtonian liquid with a uniform viscosity μ and a conductivity $\sigma > 0$. The advocated treament exploits two fundamental axisymmetric MHD flows recently obtained elsewhere and holds by essence whatever the Hartmann number $\mathrm{Ha} = aB/\sqrt{\mu/\sigma}$. It consists in determining the surface traction at the sphere boundary by inverting there a boundary-integral equation and then getting the flow velocity and pressure in the liquid by appealing to integral representations of those quantities solely in terms of the surface traction. As a result, the drag experienced by the translating sphere and the MHD flow patterns about it are given for different values of Ha. Not surprisingly, the MHD flow about the sphere is found to be very sensitive to the Hartmann number Ha.

Introduction. As sketched in Fig. 1, we examine the low-Reynolds-number axisymmetric MHD flow about a solid sphere, with a radius a and center O, translating in a quiescent and conducting Newtonian liquid at the velocity $\mathbf{U} = U\mathbf{e}_z$ parallel to the given uniform ambient magnetic field $\mathbf{B} = B\mathbf{e}_z, B > 0$.

In the absence of additional assumptions, one has to determine not only the MHD flow, with the velocity $\mathbf{u}$ and pressure p, about the sphere, but also the resulting electric field $\mathbf{E}'$ and the magnetic field $\mathbf{B}'$ in the entire liquid domain $\mathscr{D}$. Unfortunately, this problem turns to be tremendously involved [1, 2] even for a sphere since $(\mathbf{u}, p, \mathbf{E}', \mathbf{B}')$ are actually coupled through the unsteady non-linear Navier–Stokes equations and Maxwell equations. Here, for a conducting liquid having the uniform density ρ, the viscosity μ, the conductivity $\sigma > 0$ and

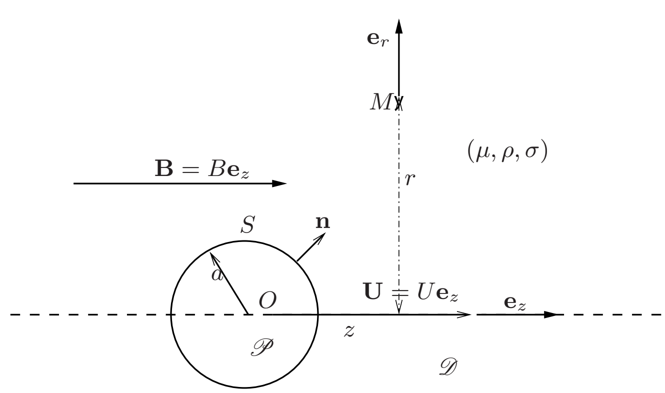

Fig. 1. A solid sphere translating in a conducting Newtonian liquid, parallel to the given uniform ambient magnetic field at the velocity $\mathbf{U} = U\mathbf{e}_z$.

the magnetic permeability $\mu_{\rm m} > 0$, the unknown coupled fields $p, \mathbf{u}, \mathbf{E}'$ and $\mathbf{B}'$ depend upon three dimensionless numbers: the magnetic Reynolds number $\mathrm{Rm} = \mu_{\rm m}\sigma|U|a$, the Reynolds number $\mathrm{Re} = \rho|U|a/\mu$ and the Hartmann number $\mathrm{Ha} = a/d$, where the length $d = (\sqrt{\mu/\sigma})/B$ is the so-called Hartmann layer thickness citeHartmann1937.

Henceforth, we take $\mathrm{Rm} \ll 1$ and assume the flow to be quasi-steady. Under those assumptions the task remains pretty complicated since $\mathbf{B}'$ has to be determined too [4]! However, if the sphere has the same magnetic permeability as the liquid and $\mathrm{Rm} \ll 1$, it appears that $\mathbf{B}' = \mathbf{B}$ [5]. Because the flow $(\mathbf{u}, p)$ is axisymmetric, and *without swirl* one also gets $\mathbf{E}' = 0$ [1, 4]. Although $\mathbf{B}' = B\mathbf{e}_z$ and $\mathbf{E}' = \mathbf{0}$, getting the MHD flow about the sphere, whatever Ha and Re, is still a challenging task. However, assuming further that $\mathrm{Re} \ll 1$ enabled [6] and [7] to *asymptotically* solve the problem for $\mathrm{Ha} \ll 1$ and $\mathrm{Ha} \gg 1$, respectively.

This work presents a new boundary approach which allows one to compute, in essence, whatever the Hartmann number $\mathrm{Ha} > 0$, the required axisymmetric MHD viscous flow $(\mathbf{u}, p)$ in the entire liquid domain and also the resulting drag exerted on the translating sphere.

1. Governing problem and challenging issues. As explained in the introduction and also illustrated in Fig. 1, we consider the axisymmetric MHD viscous flow, *without swirl* and with pressure and velocity fields p and $\mathbf{u}$ in a liquid domain $\mathscr{D}$, about a solid sphere with the radius a translating in a quiescent conducting Newtonian liquid at the velocity $\mathbf{U} = U\mathbf{e}_z$ parallel to the uniform magnetic field $\mathbf{B} = B\mathbf{e}_z$. Assuming that $\mathrm{Re} = \mu_{\rm m}\sigma|U|a \ll 1$ and neglecting all inertial effets, i.e. taking here $\mathrm{Re} = \rho|U|a/\mu \ll 1$, the flow $(\mathbf{u}, p)$ obeys the following well-posed MHD creeping flow problem

$$\mu\nabla^2\mathbf{u} = \nabla p - \sigma B^2(\mathbf{u} \wedge \mathbf{e}_z) \wedge \mathbf{e}_z \text{ and } \nabla.\mathbf{u} = 0 \text{ in } \mathscr{D}\,, \tag{1}$$

$$(\mathbf{u}, p) \to (\mathbf{0}, 0) \text{ as } |\mathbf{x}| \to \infty\,,\ \mathbf{u} = U\mathbf{e}_z \text{ on } S. \tag{2}$$

Inspecting Eqs. (1)–(2) immediately shows that the normalized flow $\mathbf{u}' = \mathbf{u}/U, p' = ap/(\mu U)$ solely depends upon the Hartmann number $\mathrm{Ha} = a/d$ which compares the sphere radius a with the Hartmann layer thickness $d = (\sqrt{\mu/\sigma})/B$ [3].

We locate each point $\mathbf{x}$ in the domain $\mathscr{D} \cup S$ by its cylindrical coordinates (r, z, θ) with $\theta \in [0, 2\pi], z = \mathbf{x}.\mathbf{e}_z$ and $r = \{|\mathbf{x}|^2 - z^2\}^{1/2} \geq 0$. Accordingly, one has $\mathbf{x} = r\mathbf{e}_r + z\mathbf{e}_z$ with the usual local unit vector $\mathbf{e}_r = \mathbf{e}_r(\theta)$ shown in Fig. 1. For the axisymmetric MHD viscous flow $(\mathbf{u}, p)$ *without swirl*, one then gets the properties $\mathbf{u}(\mathbf{x}) = u_r(r, z)\mathbf{e}_r + u_z(r, z)\mathbf{e}_z$ and $p(\mathbf{x}) = p(r, z)$ at each point $\mathbf{x}$ in the liquid. At the sphere boundary S with the unit n pointing into the liquid, this flow, with the stress tensor $\boldsymbol{\sigma}$, exerts a surface traction $\mathbf{f} = \boldsymbol{\sigma}.\mathbf{n}$ of the following form $\mathbf{f} = f_r(r, z)\mathbf{e}_r + f_z(r, z)\mathbf{e}_z$. For symmetry reasons, the sphere thus experiences a zero torque about its center O and a force $\mathbf{F}$ parallel to the sphere velocity $\mathbf{U} = U\mathbf{e}_z$. From the previous form of the traction $\mathbf{f}$, this force $\mathbf{F}$ immediately reads as

$$\mathbf{F} = \int_S \mathbf{f}\mathrm{d}S = [2\pi \int_{\mathscr{C}} f_z(P)r(P)\mathrm{d}l(P)]\mathbf{e}_z = -6\pi\mu a\lambda U\mathbf{e}_z, \tag{3}$$

where $\mathscr{C}$ is the half-circle trace of S in the $\theta = 0$ half-plane (each point P on that curve $\mathscr{C}$ has cylindrical coordinates $r(P), z(P)$ and $\theta(P) = 0$). Note that when deriving the second equality in Eq. (3), we used the relation $dS = rdld\theta$ and performed integration over θ in $[0, 2\pi]$, whereas the occurring coefficient λ is the usual so-called drag coefficient. This work considers to which extent the normalized flow $(\mathbf{u}', p')$ in the liquid domain and the resulting drag coefficient λ depend upon the Hartmann number $\mathrm{Ha} = a/d$.

2. Advocated boundary approach.

2.1. Relevant velocity and pressure integral representations. Since the axisymmetric problem (2)–(3) is linear, one can think about solving it by a boundary formulation extending thereby the usual one derived (see, among other textbooks, [8, 9]) for the usual Ha $=0$ Stokes flow case (here obtained either for a non-conducting liquid ($\sigma = 0$) or in the absence of imposed ambient magnetic field ($B = 0$)). Such boundary formulation requires to determine the fundamental axisymmetric MHD flow produced by a distribution of forces, with the strength $F_r\mathbf{e}_r + F_z\mathbf{e}_z$ and (F_r, F_z) constant, on the ring with a radius $r_0 > 0$ located in the $z = z_0$ plane. At the point $\mathbf{x}(r, z, \theta)$, the resulting MHD flow $(\mathbf{v}, q)$ is without swirl and axisymmetric. It also has a velocity field $\mathbf{v}(\mathbf{x}) = v_r(r, z)\mathbf{e}_r + v_z(r, z)\mathbf{e}_z$ and the pressure field $q(\mathbf{x}) = q(r, z)$ recently obtained in [11]. Actually, the latter paper successively builds two fundamental axisymmetric MHD flows (associated with the choices $F_r = 0$ and $F_z = 0$) by angular integration of the three-dimensional solution recently obtained in [10] for the fundamental flow and electric potential produced in the liquid by a point source with a given strength. For the associated pretty-involved steps and formulae for the resulting flow velocity (axial and radial) components and pressure, the reader is directed to [11]. Introducing $M(r, z)$ and $M_0(r_0, z_0)$ in the half $\theta = 0$ plane, taking the indices α and β in $\{r, z\}$ and adopting henceforth the usual tensor summation convention, the results detailed in [11] are

$$v_\alpha(\mathbf{x}) = \frac{1}{8\pi\mu} G_{\alpha\beta}(M, M_0)F_\beta,\; q(\mathbf{x}) = \frac{1}{8\pi} P_\beta(M, M_0)F_\beta \text{ for } M \neq M_0 \;, \tag{4}$$

with the so-called Green tensor velocity components $G_{\alpha\beta}(M, M_0)$ and the Green pressure vector components $P_\beta(M, M_0)$ given in terms of $(z - z_0, r, r_0, d)$ in [11], but not reproduced here for the sake of conciseness. Extending the treatment presented in [9] for the Ha $= 0$ Stokes flow case made it possible to derive the basic integral representations of single-layer types for the required axisymmetric flow velocity $\mathbf{u}$ and pressure p satisfying Eq. (1), the far-field behaviour (2)*and also the constant* velocity boundary condition (2). Curtailing the details which will be displayed elsewhere, one actually arrives at the relations

$$u_\alpha(\mathbf{x}) = -\frac{1}{8\pi\mu}\int_{\mathscr{C}} G_{\alpha\beta}(M, P)f_\beta(P)r(P)\mathrm{d}l(P) \text{ for } \mathbf{x} \in \mathscr{D} \cup S, \tag{5}$$

$$p(\mathbf{x}) = -\frac{1}{8\pi}\int_{\mathscr{C}} P_\beta(M, P)f_\beta(P)r(P)\mathrm{d}l(P) \text{ for } \mathbf{x} \in \mathscr{D}. \tag{6}$$

One should note that we here obtain the above single-layer representations (5)–(6) because of the very specific form of the velocity boundary condition (2) which makes it possible to prove that the additional double-layer terms also arising in general on the right-hand sides of Eqs. (5) and (6) vanish for the present problem. In [11] it has been shown that $G_{r\beta}(M, P) = 0$ for $r = 0$. Substituting that property in Eq. (5) yields $u_r(\mathbf{x}) = 0$ for $\mathbf{x}$ located on the $(O, \mathbf{e}_z)$ problem axis of revolution (as this must be the case for the regular axisymmetric velocity field $\mathbf{u}$). From Eqs. (5)–(6) it is clear that is it sufficient to gain the traction $\mathbf{f}(P) = f_\beta(P)\mathbf{e}_\beta$ on the half-circle contour $\mathscr{C}$ in order to subsequently compute the flow in the entire liquid domain.

2.2. Key coupled boundary-integral equations. The required traction $\mathbf{f}$ is gained by enforcing the no-slip condition $\mathbf{u} = U\mathbf{e}_z$ at the sphere boundary. Using

Eq. (5) yields the following coupled boundary-integral equations of the first kind

$$\int_{\mathscr{C}} G_{rr}(M,P) f_r(P) r(P) \mathrm{d}l(P) + \int_{\mathscr{C}} G_{rz}(M,P) f_z(P) r(P) \mathrm{d}l(P) = 0, \tag{7}$$

$$\int_{\mathscr{C}} G_{zr}(M,P) f_r(P) r(P) \mathrm{d}l(P) + \int_{\mathscr{C}} G_{zz}(M,P) f_z(P) r(P) \mathrm{d}l(P) = -8\pi\mu U \tag{8}$$

for M on $\mathscr{C}$.

Of course, for symmetry reasons, one has $f_r(P) = 0$ at the points P located on the problem axis of revolution $(O, \mathbf{e}_z)$, while at M on $\mathscr{C}$ satisfying $r(M) = 0$ one solely requires Eq. (8) (relation (7) being satisfied from the previous property $G_{r\beta}(M,P) = 0$).

2.3. Numerical implementation. The boundary-integral equations (7)–(8) have been numerically inverted by carefully evaluating each influence coefficient $G_{\alpha\beta}(M, M_0)$ (see the procedure described in details in [11]) and by splitting the half-circle, with the unit radius $a = 1$ and center O into 16 curved 3-node quadratic boundary elements with equal length. The discretized coupled boundary-integral equations (7)–(8) are then numerically enforced at the resulting 31 nodal points located off the sphere axis of revolution $(O, \mathbf{e}_z)$. Of course, only Eq. (8) is imposed at the two remaining nodal points located on the sphere axis of revolution (as previously pointed out, Eq. (7) is trivially satisfied at those nodes). It has been numerically found that taking $N_{\mathrm{e}} = 16$ curved boundary elements is quite sufficient to ensure a five-digit accuracy for the drag coefficient λ given by Eq. (3) in the entire range Ha ≤ 30. For instance, at Ha $= 30$, one obtains $\lambda = 11.837020$ for $N_{\mathrm{e}} = 16$ and $\lambda = 11.837016$ for $N_{\mathrm{e}} = 32$.

3. Numerical results. This section presents numerical results both for the drag coefficient λ and for the flow patterns about the translating sphere.

3.1. Drag coefficient. Let us first consider the drag coefficient λ defined by Eq. (3). This basic quantity has been asymptotically approximated at a small Hartmann number Ha in [6] by a procedure quite different from the boundary one advocated in this paper. More precisely, [6] predicts that

$$\lambda \sim \lambda_a = 1 + 3\mathrm{Ha}/8 + 7\mathrm{Ha}^2/960 - 43\mathrm{Ha}^3/7680 \tag{9}$$

for Ha small enough.

As shown in Table 1, our computations are in excellent agreement with Eq. (9) for Ha in the range [0,1].

In contrast to the quite different procedure worked out in [6], the present boundary approach makes it possible to investigate the case of larger values of the Hartmann number Ha. The drag coefficient has thus been computed for Ha ≤ 30. The results are plotted in Fig. 2 solely for Ha ≤ 10 in order to clearly reveal the range of validity of the approximation (9) which turns out to be very good for Ha ≤ 2. As also shown in Fig. 2, the drag coefficient is found to increase (nearly in a linear fashion) versus Ha for large Ha.

Table 1. Comparisons between the computed drag coefficient λ and its asymptotic estimate λ_a (see Eq. (9)).

Ha	0.01	0.1	0.3	0.5	0.7	1
λ	1.00381	1.03763	1.11310	1.18886	1.26479	1.37884
λ_a	1.00375	1.03757	1.11301	1.18862	1.26415	1.37669

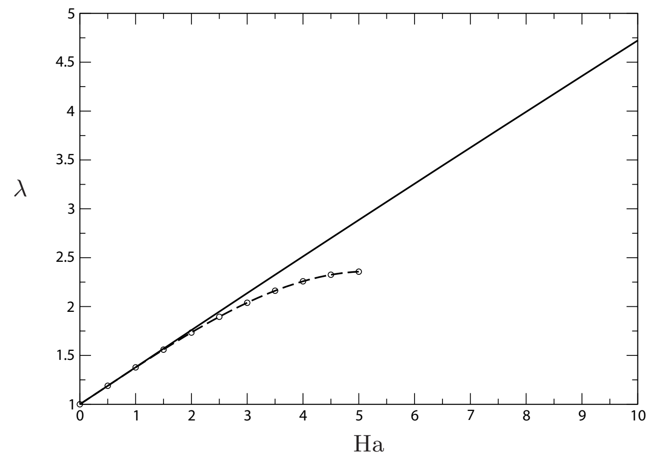

Fig. 2. Computed (solid line) and asymptotic (dashed line with open circles obtained using Eq. (9)) drag coefficient.

3.2. Flow patterns. We now investigate the axisymmetric MHD flow sensitivity to the Hartmann number Ha. This is done by computing in the liquid domain the normalized pressure p' and the normalized radial and axial velocity components u'_r and u'_z defined as

$$p' = ap/(\mu U),\ u'_r = u_r/U,\ u'_z = u_z/U. \tag{10}$$

Since the flow is axisymmetric, those quantities are drawn only in the $\theta = 0$ half-plane using the normalized coordinates $r' = r/a \geq 0$ and $z' = z/a$. Similarly, the flow streamlines are provided in the same half-plane.

For Ha = 0, one retrieves the well-known problem of the Stokes flow about a sphere with a radius a translating in a Newtonian liquid (either insulating or conducting) in the absence of a magnetic field for which the previous quantities are analytically known (see, for instance, [9]). Using the above normalized coordinates (r', z') and setting $\rho' = \{r'^2 + z'^2\}^{1/2}$, one actually gets at $M = 0$,

$$u'_r = \frac{3z'r'}{4\rho'^3}\left[1 - \frac{1}{\rho'^2}\right],\ u'_z = \frac{3}{4\rho'}\left[1 + \frac{1}{3\rho'^2}\right] + \frac{3z'^2}{4\rho'^3}\left[1 - \frac{1}{\rho'^2}\right],\ p' = \frac{3z'}{2\rho'^3}. \tag{11}$$

The results (11) are further employed to draw the figures at Ha $= 0$, whereas other figures for flow patterns at Ha > 0 are numerically obtained, outside the sphere boundary, from the integral representations (5)–(6). The numerical results agree with the symmetries $u'_r(r', z') = -u'_r(r', -z'), u'_z(r', z') = u'_z(r', -z')$ and $p'(r', z') = -p'(r', -z')$ which can be easily established from the governing problem (1)–(2).

The dependence versus Ha of the normalized radial velocity component u'_r is first investigated by plotting in Fig. 3 the isolevel contour curves of this quantity for Ha $= 0, 0.1, 1, 5, 10, 20$ for the points (r', z') in the liquid domain such that $r' \leq 3$ and $-3 \leq z' \leq 3$. For a given value of Ha, the normalized velocity component u'_r vanishes (due to the no-slip condition) on the sphere half-contour $\mathscr{C}$, remains small in the liquid domain except for the two pockets (in which, say, $|u'_z| \geq 0.1$) located close above $\mathscr{C}$ and also quickly decays away from $\mathscr{C}$ *in all directions* as $\rho' = \{r'^2 + z'^2\}^{1/2}$ becomes large. As Ha increases, $|u'_r|$ is seen to decrease in the liquid, whereas the two previous pockets shrink and clearly approach $\mathscr{C}$ near its $(r', z') = (1, 0)$ point.

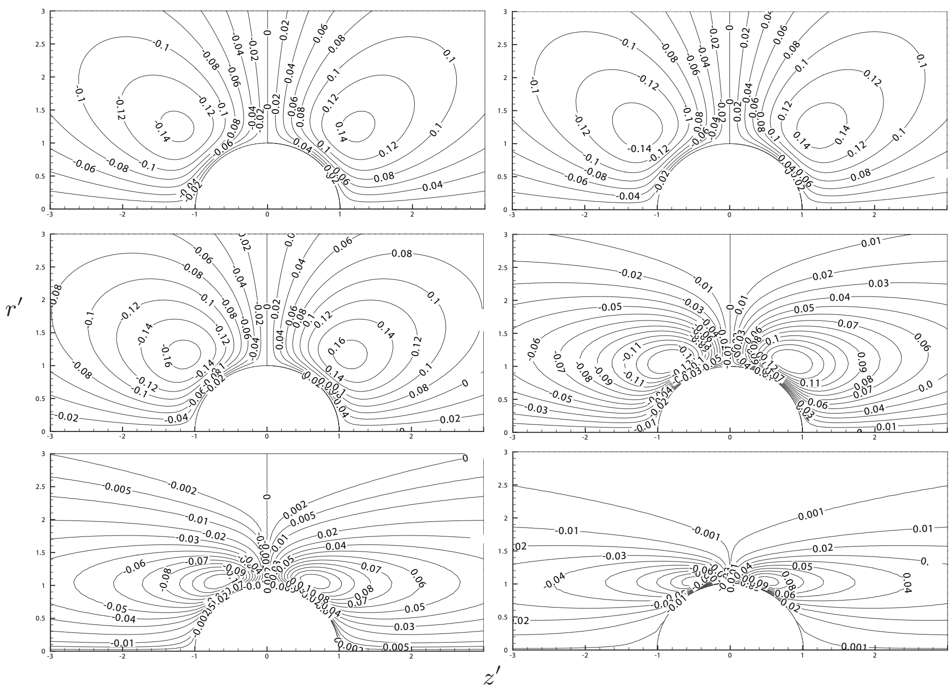

Fig. 3. Isolevel lines for the normalized radial velocity component u'_r for different values of the Hartmann number with the following choices: Ha $= 0$ (Stokes flow) top left; Ha $= 0.1$ top right; Ha $= 1$ middle left; Ha $= 5$ middle right; Ha $= 10$ bottom left, and Ha $= 20$ bottom right.

The case of the normalized axial velocity component u'_z is illustrated in Fig. 4 for the same values of Ha. Not surprisingly, for a prescribed value of Ha one obtains $u'_z = 1$ on $\mathscr{C}$ (from the no-slip boundary condition) and also $0 < u'_z < 1$ decreases away from $\mathscr{C}$ as ρ' increases. However, in contrast to the previous case of the radial velocity u'_r, such a decay clearly depends upon the direction in which one goes away from $\mathscr{C}$! More precsisely, there is a large domain near the $r' = 0$ axis of symmetry in which u'_z remains of significant value, say, $u'_r \geq 0.5$. In a sense, one can thus speak of wakes in the upstream ($z' < 0$) and downstream ($z' > 0$) directions for the quantity u'_z. As Ha increases, the previous trends are still valid, with a quicker decay of u'_r away from the $r' = 0$ axis of symmetry and, in contrast, previous wakes which now extend away from the sphere in the $r' = 0$ upstream and downstream directions.

The trends observed in Figs. 3 and 4 when Ha increases suggest that, as Ha becomes very large, the normalized velocity $\mathbf{u}'$ is nearly zero exept in the liquid located in two 'upstream' ($z < 0$) and 'downstream ($z > 0$) wakes (located in the normalized "tube" $r' \leq 1$), where it should tend to the normalized sphere translational velocity $\mathbf{e}_z$. This conclusion agrees with the asymptotic prediction proposed in [7] for the MHD axisymmetric flow about a body of revolution translating at large Ha parallel to both its axis of symmetry and to the prescribed ambient uniform magnetic field. More precisely, from [7], those wakes actually extend upstream and downstream up to $|z'| \leq O(\text{Ha})$. In accordance with that prediction, the computed streamlines (which are of course the same for both velocity fields $\mathbf{u}$ and $\mathbf{u}'$) presented in Fig. 5 clearly tend to become parallel to the $r' = 0$ axis in the wakes as Ha increases. Outside those wakes, the streamlines are less parallel to the $r' = 0$ axis as Ha increases, but the normalized velocity magnitude $|\mathbf{u}'|$ there collapses as Ha increases.

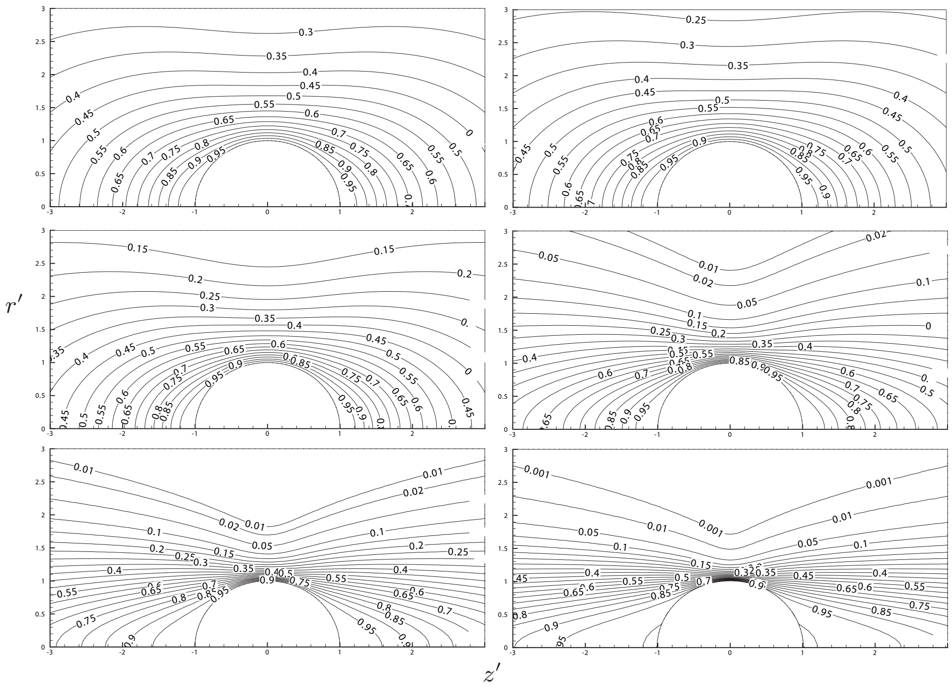

Fig. 4. Isolevel lines for the normalize axial velocity component u'_z for different values of the Hartmann number with the following choices: Ha $= 0$ (Stokes flow) top left; Ha $= 0.1$ top right, Ha $= 1$ middle left; Ha $= 5$ middle right; Ha $= 10$ bottom left, and Ha $= 20$ bottom right.

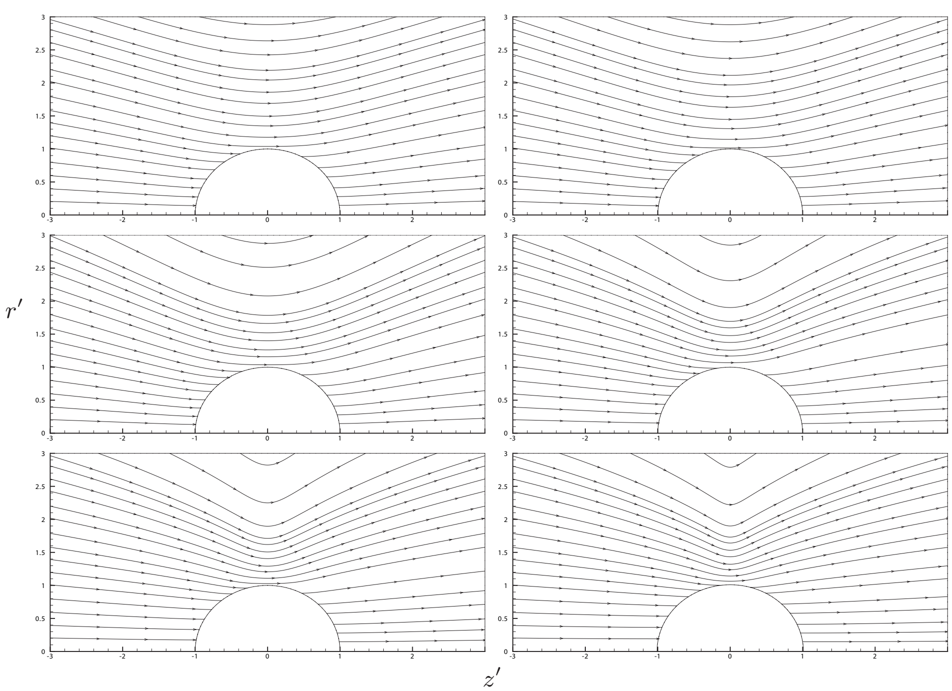

Fig. 5. Flow streamlines for different values of the Hartmann number with the following choices: Ha $= 0$ (Stokes flow) top left, Ha $= 0.1$ top right, Ha $= 1$ middle left, Ha $= 5$ middle right, Ha $= 10$ bottom left, and Ha $= 20$ bottom right.

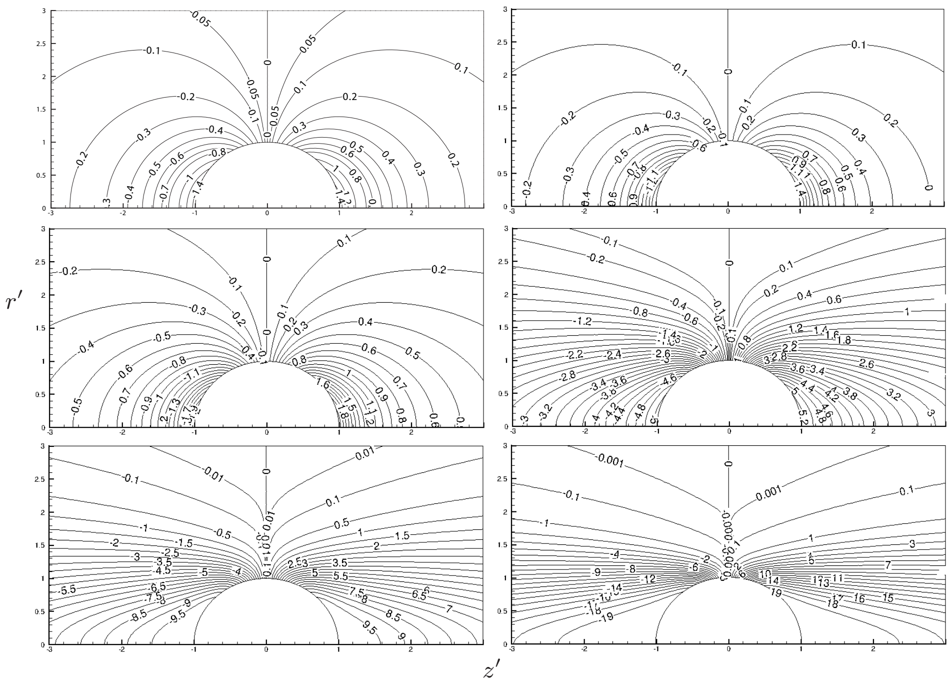

Fig. 6. Normalized pressure p' for different values of the Hartmann number with the following choices: $\mathrm{Ha} = 0$ (Stokes flow) top left, $\mathrm{Ha} = 0.1$ top right, $\mathrm{Ha} = 1$ middle left, $\mathrm{Ha} = 5$ middle right, $\mathrm{Ha} = 10$ bottom left, and $\mathrm{Ha} = 20$ bottom right.

Finally, we consider the normalized pressure p' by plotting in Fig. 6 the associated isolevel contour curves. With a given Ha, the pressure field p' quickly or sowly decays away from $\mathscr{C}$ normal to or along the $r' = 0$ axis, respectively. Note also that the pressure exhibits large changes on and near $\mathscr{C}$, especially in the vicinity of the $(r', z') = (1, 0)$ point. As Ha increases, p' becomes very small except again in the previous wakes in which it reaches large values and its magnitude increases with Ha.

4. Conclusions. A new boundary formulation has been proposed to accurately and efficiently determine the viscous axisymmetric MHD flow about a solid sphere translating at the velocity $\mathbf{U}$ in a conducting and quiescent Newtonian liquid, parallel to a prescribed uniform ambient magnetic field. As a consequence, it has been possible to compute the drag experienced by the sphere and the flow about it for several values of the Hartmann number Ha. The results clearly reveal that the drag increases with Ha nearly in a linear fashion, whereas the flow patterns (velocity components, streamlines and pressure) are found to deeply depend upon Ha. The computations also suggest that for $\mathrm{Ha} \gg 1$ the MHD flow $(\mathbf{u}, p)$ will be confined in a liquid "tube" in which $\mathbf{u} \sim \mathbf{U}$ and p is large. Those findings agree well with the very first-order prediction derived in [7] for $\mathrm{Ha} \gg 1$ and suggest to build in the future a high-order asymptotic analysis of the problem for large Ha. Such a challenging task requires additional efforts and is thus postponed to another work.

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