

The Multimodal Dynamics of a Walnut Tree: Experiments and Models

M. Rodriguez

Department of Mechanics, LadHyX,
Ecole Polytechnique, CNRS,
91128 Palaiseau, France;
INRA, UMR 547 PIAF,
63100 Clermont-Fd, France

S. Ploquin

B. Moulia

e-mail: moulia@clermont.inra.fr

INRA, UMR 547 PIAF,
63100 Clermont-Fd, France;
Université Blaise Pascal,
UMR 547 PIAF,
63100 Clermont Fd, France

E. de Langre¹

Department of Mechanics, LadHyX,
Ecole Polytechnique, CNRS,
91128 Palaiseau, France
e-mail: delangre@ladhyx.polytechnique.fr

The dynamics of a walnut tree is investigated in order to better understand the mechanical interaction of trees with wind. Experimental data on the vibrational modes of the tree are obtained, using pull and release tests. These results are then used to validate a previous analytical approach that predicts the organization of modal frequencies as a function of two allometry parameters describing the branched tree geometry. In addition to these experimental results, vibration modes are also obtained from a finite element computation using a detailed digitization of the tree geometry. The comparison between experiments, computation, and the simple analytical approach confirm the specific organization of modes of such branched trees, with a high modal density and a spatial localization. Then the possible biological importance of this organization and the potential biomimetic applications are outlined. [DOI: 10.1115/1.4005553]

1 Introduction

The dynamics of plants is important to understand their interaction with their mechanical environment [1]. Plants and wind mechanical interactions during windstorms are responsible for large windbreaks and windthrows and, consequently, can represent substantial ecological and economical costs [2]. Wind-induced movement is also a key mechanism in the plant acclimation to the wind climate through growth regulation [3]. Plant dynamics is also important in agricultural practices. The forced dynamical excitation of trees is commonly used in fruit harvesting [4]. Finally, in computer graphics, modeling plant dynamics is of

great interest for real-time animation of large virtual scenes [5]. All these issues underline the need for a good understanding and modeling of plant dynamics.

From a mechanical point of view there are significant differences between single stem plants, such as wheat, and trees. Because of their geometry, simple plants have a behavior similar to that of a beam in bending, and the canopy they form have a specific coupling with the wind [6]. Trees, on the contrary, display in most cases complex branched architectures with several orders of branching. This branched organization makes the tree a particular mechanical structure compared to the usual mechanical engineering design. We shall focus here on the case of an individual branched tree. Previous works on trees have shown that considering dynamical aspects are essential to predict the critical flow speed for windbreak and windthrow [7,8].

While investigating the role of the whole tree architecture on its dynamics under wind from measurements on the trunk, multiple frequencies were sometimes found in the spectra (see Ref. [9]). It has been then argued that this high frequency content was due to a dynamical interaction between the branches and the trunk, as in the classical mass dampers mechanism. Yet, no branches of high orders were instrumented to confirm this hypothesis. Experimental data on olive trees with accelerometers in the main branches only also showed a high density of modes [4].

In an attempt to understand the dynamical organization of trees, computations of vibration modes from digitized tree geometries have been carried out in Refs. [10,11]. Results have shown a concentration of modes in terms of modal frequencies and higher frequency modes with displacements localized on branches. More generally, multiple branched systems are known to have an interesting behavior in terms of the organization of frequencies [12] and of localization of deformation [13].

To understand the multimodal dynamics of trees and to explain quantitatively the role of their geometry on their dynamical characteristics, the case of an idealized symphydial tree (identical branches at each branching) has been analyzed in Ref. [14]. The description of the tree architecture has been simplified using the two standard parameters: (i) the branching ratio λ to describe the reduction of diameter through branching, $D_{N+1}/D_N = \sqrt{\lambda}$, and (ii) the slenderness exponent β to describe the relationship between length L and diameter D in branch segments of the tree, $D \sim L^\beta$. Using this approach, the tree geometry can be described in terms of subsets. A subset of order N consists in a part of the tree that starts from a branch, which is separated from the trunk by $N-1$ branching (see Fig. 1). Using finite element computations, modes in such an idealized tree are found to be localized in subsets of the tree and can, therefore, be labeled according to the order of the subset in which their displacements are localized [14] (see Fig. 1).

Based on self-similarity arguments, a scaling law has been derived in Ref. [14] that relates the modal frequencies f_N of modes with the subset order N using only the two geometrical parameters λ and β . The argument is based on the fact that a subset of order N is identical to the whole tree but for an allometric change in both length and diameter scales, through λ and β . As the frequency of a bending mode varies like dl^{-2} where l is the scale of lengths and d is the scale of diameters, the frequencies of modes of different subsets were found to be related by

$$f_N/f_1 = \lambda^{(N-1)(\beta-2)/\beta} \quad (1)$$

This simple law was shown to give a good approximation of the computed modes in a FEM analysis of a 3D digitized real tree.

The aim of the present paper is to test this scaling law in comparison with experimental results. In Sec. 2, modal frequencies are obtained from experiments on a young walnut. They are compared in Sec. 3 with those from the scaling law. In Sec. 4 a comparison with results from finite-element

¹Corresponding author.

Contributed by the Applied Mechanics Division of ASME for publication in the JOURNAL OF APPLIED MECHANICS. Manuscript received November 25, 2010; final manuscript received September 8, 2011; accepted manuscript posted January 30, 2012; published online May 11, 2012. Assoc. Editor: Alexander F. Vakakis.

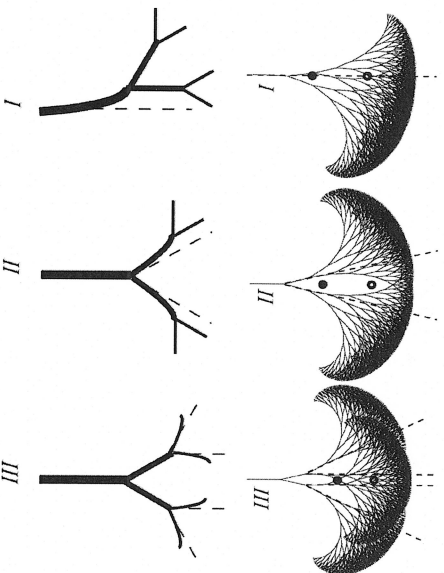
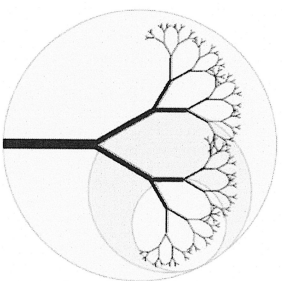


Fig. 1 Idealized branched tree. Top: a branched tree and self-similar subsets. Center: computed modes [14]. Bottom: schematic representation of the mode shapes corresponding to groups of different orders (redrawn from [14]).

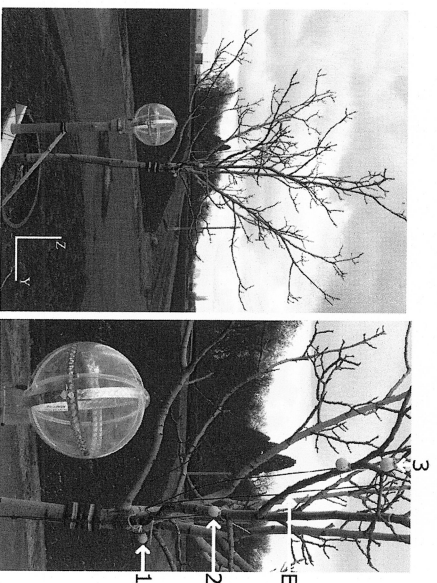


Fig. 2 Experimental setup. Left: the young walnut seen from x-direction. Right: positions of the excitation (point E) and location of sensors (point S1, S2, and S3).

computations is also given to help understand the dynamics. Finally we discuss in Sec. 5 the biomechanical significance of these results.

2 Experiments

2.1 Material and Methods. A walnut tree (*Juglans regia* L.), grown in a large pot, has been located in a flat open-field area (see Fig. 2(a)). The tree was 10-years-old, 4.2 m high, with a trunk of 7.7 cm in diameter at 1.30 m high and up to five consecutive levels of branching from the trunk to the tips. The movements were recorded using displacement sensors connected to a 3D

Pollhemus Fastrack Long Range (Colchester, VT, USA), which generates a magnetic field and then allows us to determine locations of sensors in the 3D-space field [15]. The sensors spatial positions were recorded at 30 Hz, and the signal is consistent with millimeter resolution, probably in relation to the very low level of magnetic perturbation in the field. Three displacement sensors were positioned on three following segments of the tree: one on the trunk, one on a second order branch and one on a fourth order branch (see Fig. 2). Excitation was carried out by pull and release tests. Other tests with impact or wind excitation gave similar results and are not presented here for the sake of brevity.

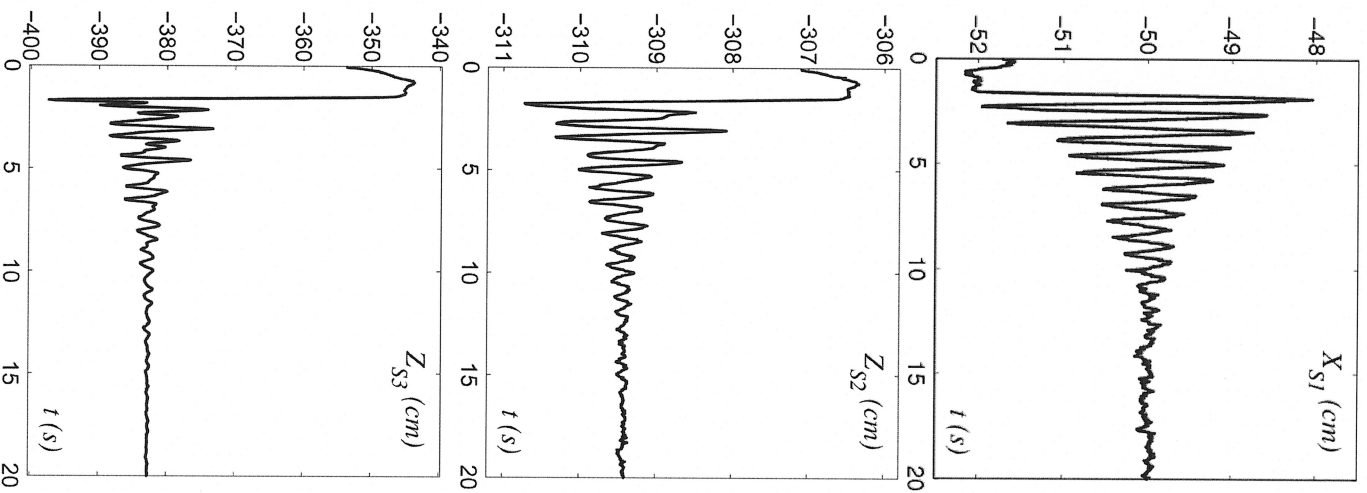


Fig. 3 Displacements at sensors S1, S2, and S3 during a pull and release test

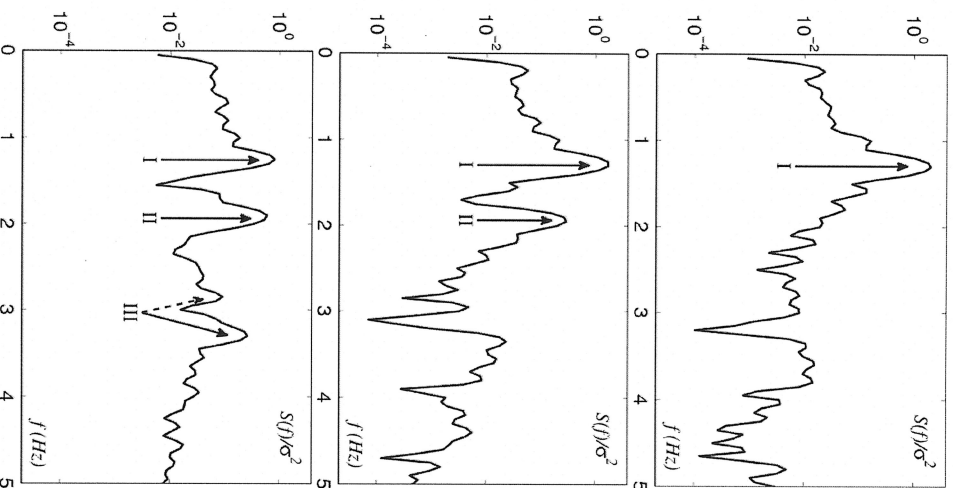


Fig. 4 Normalized PSD of responses for each sensor: S1 (top), S2 (center), and S3 (bottom). Peaks are identified with arrows, secondary peaks with dashed arrows, and labeled according to the frequency range. Higher modes are only seen in the displacement of sensors in the branches.

2.2 Results. Figure 3 presents typical results of a pull and release test applied at the position (E) in the vertical z-direction, corresponding to an excitation on a third order branch. This excitation resulted mainly in displacements in the x-direction for the sensor S1 and in the z-direction for the sensors S2 and S3. Figure 3 shows the temporal responses recorded on the sensors. The time evolution recorded on sensor S1 exhibits typical damped sinusoidal oscillations, while responses recorded on sensors S2 and S3 showed multimodal time evolutions.

In order to analyze the differences between responses recorded at different sensors locations and to get more information about the tree modal characteristics, the power spectral density of responses were computed (see Fig. 4). On sensor S1, one main frequency, labeled I, is found at 1.3 Hz. On sensor S2, the peak at 1.3 Hz is also found, with an additional peak near 2 Hz, labeled II in Fig. 4. On sensor S3, additional peaks between 2.8 and 3.3 Hz are found. Table 1 sums up the identification of frequency peaks on the sensors. Also in Table 1 are shown the rms values of these displacements. They are found to increase with branch order, i.e., from sensor S1 to sensor S3. These experimental results give clear evidence that the higher the branch order, the higher the modal frequencies localized on it.

3 The Organization of Frequencies

3.1 Parameters Describing the Walnut Geometry. The walnut geometry has been digitized using a static version of the

Table 1 RMS amplitude and dominant frequencies of the displacement of each sensor

Sensor	RMS (cm)	Frequencies (Hz)		
		I	II	III
S1	0.95	1.3		
S2	1.35	1.3	1.9	
S3	5.25	1.3	1.9	2.8-3.3

magnetic sensor tracking, called 3D digitizing [16]. It consisted in recording for each segment, the spatial coordinates of its extremities, its diameters, and its connectivities. The resulting architecture is shown in Fig. 5. The two parameters β and λ used to describe the tree geometries are then statistically determined from the walnut data using orthogonal regression analysis. A good linear fit was observed with coefficients of determination $R^2 = 0.89$ and $R^2 = 0.82$, respectively, for the slenderness exponent and the branching ratio. The slenderness exponent was found to be $\beta = 0.82$ with a 10% confidence level equal to $0.72 \leq \beta \leq 0.92$, and the branching ratio was $\lambda = 0.31$ with a 10% confidence level equal to $0.27 \leq \lambda \leq 0.35$.

3.2 Comparison Between Scaling Law and Experiments. Using the scaling law given in Eq. (1) with the geometrical parameters derived above, the relation between frequencies of different groups of modes can be predicted. In Fig. 6, the nominal frequency of each group, normalized by the frequency of the first mode is shown as a function of the group number N . Only the integer values of N are relevant. Taking into account the confidence interval on β and λ , yields a confidence interval on the predicted curve, here shown in gray. On the same figure are shown the normalized frequencies taken from the experiments, as given in Table 1. The prediction of the scaling law overestimates the experimental data, but the qualitative evolution is correctly predicted. Note also that the experiments and the model are also consistent in the fact that the increasing group number corresponds to modes localized in subsets of the tree.

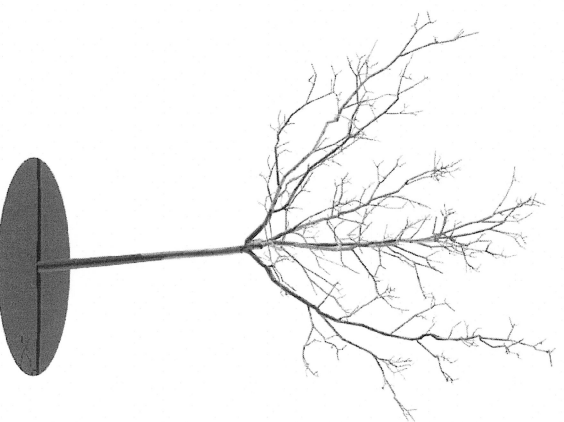


Fig. 5 Digitized geometry of the walnut tree used for the computation of the allometry parameters λ and β in Sec. 3.1 and for the finite element computations in Sec. 4

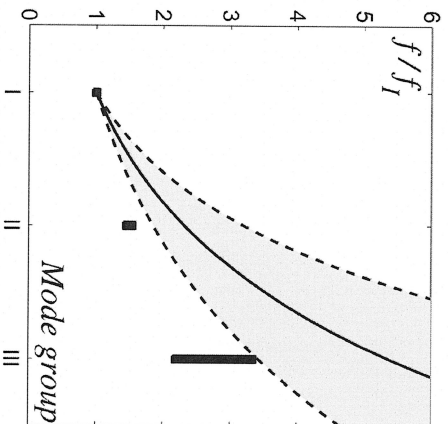


Fig. 6 Normalized frequencies of modes of the walnut tree, as a function of the group of modes. (■) Experimental data from Table 1. (—) Prediction using Eq. (1) with λ and β derived from the actual tree geometry. The gray area corresponds to the 90% confidence interval on the geometrical parameters.

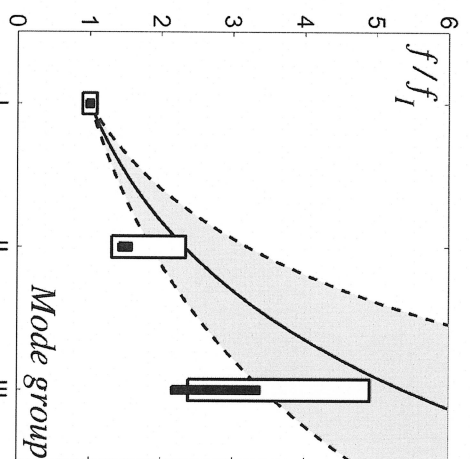


Fig. 8 Frequency ratio of modes of different groups from experiments (■), finite-element computations, (□) and from the prediction using the allometry parameters (—)

4 Finite Element Analysis

To further understand the dynamics of the system, the modal frequencies are now obtained from a finite element computation using the full digitized geometry of the tree and Euler beam model for transverse deformations, following Ref. [14]. Beam cross-sections were assumed to be circular, with a variable diameter along the beam when available. Connections between segments were set as rigid and the root anchorage is modeled as a perfect clamping. The green-wood material properties, density $\rho = 805 \text{ kg} \cdot \text{m}^{-3}$, Young modulus $E = 11.3 \text{ GPa}$, and Poisson ratio $\nu = 0.38$, as in Ref. [14], were assumed to be uniform over all branches of the tree. The finite element freeware CASTEM v. 3M [17] was used to compute the modes. The forty lowest modes were determined from the digitized geometry, made of 4653 elements. The mode frequencies were then grouped according to the main localization of the modal strain energy and tagged by the order of the branches where deformations are mainly localized (see Fig. 7). Forty frequencies are found between one and five times the first mode frequency, showing a very high density of modes. These computed modal frequencies are now compared to

frequencies obtained in the experiments (see Fig. 8). Note that the computed frequencies cover a larger range than the experimental frequencies as the latter only show the few modes excited in the experiments. The experimental and numerical results are quite consistent. This shows that a simple finite element analysis with a beam model and homogeneous material properties allow us to derive good approximations of the modal organization in a branched tree with a complex geometry. It also shows that the difference found between the model of Eq. (1) and the experiments comes mostly from the oversimplification of the geometry in the model. A finer description, as in the finite element model, allows a more accurate recovery of the frequencies.

5 Conclusion

The comparison between experiments, model, and FEM computation confirms the specific organization of modes of branched trees, with a high modal density and a spatial localization of modes. This specific organization is mostly related to the geometry of the tree resulting from its growth and can be approximated through a simple analytical approach based on two allometric parameters. An important question remains as to the biological importance of the particularly dense modal content of branched trees. It has been argued that a dense modal content may allow exchanges of energy between modes and would, thereby, be a possible way to limit response to wind [9,18]. This exchange of energy may occur through nonlinear effects such as large displacements in the tree, or impact and friction between branches, or nonlinear aerodynamic damping (see Ref. [1]). If this modal organization is really improving the resistance to extreme wind events and, thereby, possibly the fitness of plants to their environment, then evolution is likely to have selected a regulation in the growth process that allows keeping it. In fact, recent work has shown that wind does affect the geometrical parameters defined in this paper (see for instance Ref. [19]). By doing so, a tree may tune its modal properties so that the frequencies remain densely organized near the first frequency [20].

In a distinct perspective, biomimetic applications can be undertaken. By designing a structure with an adequate branching pattern, using Eq. (1), a high level of damping on the first mode may be expected by transfer of energy to higher modes [21].

Acknowledgment

We would like to acknowledge the help of Boris Adam and Nicolas Dones in designing the experimental setup. Moreover we

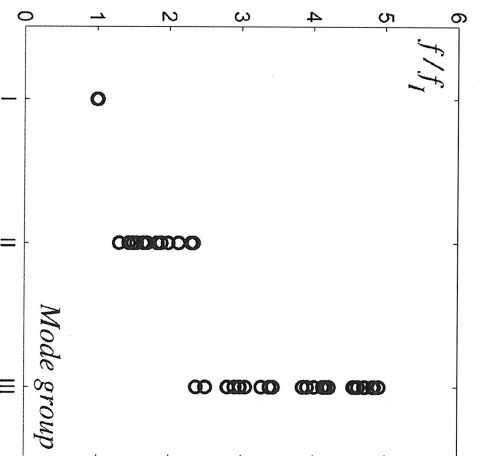


Fig. 7 Normalized computed eigenfrequencies of the walnut. Frequencies are grouped according to the main localization of modal deformations.

thank Yves Brunet, Pascal Hémon, and Lionel Reveret for fruitful discussions. This work was supported by ANR Grant No. ANR-06-BLAN-0210-02 "Chêne-Roseau" involving INRA, INRIA, and Ecole Polytechnique. We also acknowledge CNRS and INRA for the Ph.D. scholarship of Mathieu Rodriguez.

References

- [1] de Langre, E., 2008, "Effects of Wind on Plants," *Annu. Rev. Fluid Mech.*, **40**, pp. 141–168.
- [2] Gardiner, B., and Quine, C., 2000, "Management of Forests to Reduce the Risk of Abiotic Damage – A Review With Particular Reference to the Effects of Strong Winds," *Forest Ecol. Manage.*, **135**, pp. 261–277.
- [3] Moulla, B., Coutand, C., and Lenne, C., 2006, "Posture Control and Skeletal Mechanical Acclimation in Terrestrial Plants: Implications for Mechanical Modeling of Plant Architecture," *Am. J. Bot.*, **93**, pp. 1477–1489.
- [4] Castro-García, S., Blanco-Roldán, G., Gil-Ribes, J., and Agüera-Vega, J., 2008, "Dynamic Analysis of Olive Trees in Intensive Orchards Under Forced Vibration," *Trees*, **22**(6), pp. 795–802.
- [5] Dieci, J., Rodriguez, M., Baboud, L., and Reveret, L., 2009, "Wind Protection Basis for Real-Time Animation of Trees," *Comput. Graph. Forum*, **28**(2), pp. 533–540.
- [6] Py, C., de Langre, E., and Moulla, B., 2006, "A Frequency Lock-In Mechanism in the Interaction Between Wind and Crop Canopies," *J. Fluid Mech.*, **568**, pp. 425–449.
- [7] Mayer, H., 1987, "Wind-Induced Tree Sways," *Trees*, **1**(4), pp. 195–206.
- [8] Gardiner, B., Petola, H., and Kellomäki, S., 2000, "Comparison of Two Models for Predicting the Critical Wind Speeds Required to Damage Coniferous Trees," *Ecol. Modell.*, **129**(1), pp. 1–23.
- [9] James, K., Harris, N., and Ades, P., 2006, "Mechanical Stability of Trees Under Dynamic Loads," *Am. J. Bot.*, **93**, pp. 1522–1530.
- [10] Fournier, M., Rogier, P., Costes, E., and Jaeger, M., 1993, "A Mechanical Model of Wind-Induced Natural Sways of Trees Related to Branching Patterns," *Annales des Sciences Forestières*, **50**, pp. 401–412.
- [11] Sellier, D., and Fourcaud, T., 2009, "Crown Structure and Wood Properties: Influence on Tree Sway and Response to High Winds," *Am. J. Bot.*, **96**(5), pp. 885–896.
- [12] Leung, A., 2010, "Dynamic Substructure Method for Elastic Fractal Structures," *Comput. Struct.*, **89**, pp. 302–315.
- [13] Sapoval, B., Félix, S., and Flöche, M., 2008, "Localisation and Damping in Resonators With Complex Geometry," *Eur. Phys. J. Spec. Top.*, **161**(1), pp. 225–232.
- [14] Rodriguez, M., de Langre, E., and Moulla, B., 2008, "A Scaling Law for the Effects of Architecture and Allometry on Tree Vibration Modes Suggests a Biological Tuning to Modal Compartmentalization," *Am. J. Bot.*, **95**(12), pp. 1523–1537.
- [15] Rudnicki, M., and Burns, D., 2006, "Branch Sway Period of Four Tree Species Using 3D Motion Tracking," *Proceedings of the 5th Plant Biomechanics Conference*, pp. 25–31.
- [16] Shinoquet, H., and Rivet, P., 1997, "Measurement and Visualization of the Architecture of an Adult Tree Based on a Three-Dimensional Digitizing Device," *Trees*, **11**, pp. 265–270.
- [17] Verpeaux, P., Charas, T., and Millard, A., 1988, *Cartem 2000: Une approche moderne du calcul des structures*, Plurallis, Paris, France, pp. 261–271.
- [18] Spatz, H., Bruchert, F., and Pfisterer, J., 2007, "Multiple Resonance Damping or How do Trees Escape Dangerously Large Oscillations?," *Am. J. Bot.*, **94**, pp. 1603–1611.
- [19] Watt, M., Moore, J., and McKinlay, B., 2005, "The Influence of Wind on Branch Characteristics of *Pinus Radiata*," *Trees*, **19**(1), pp. 58–65.
- [20] Moulla, B., Der Loughian, C., Bastien, R., Martin, L., Rodriguez, M., Gourelleau, D., Barbact, A., Badel, E., Franchel, J., Lenne, C., Ročekel-Drevet, P., Allain, J.-M., Frachisse, J.-M., de Langre, E., Coutand, C., Fournier-Lablan, N., and Julien, J.-L., 2011, "Integrative Mechanobiology of Growth and Architectural Development in Changing Mechanical Environments," *Mechanical Integration of Plant Cells and Plants*, Springer Series: Signaling and Communication in Plants, P. Wojtaszek, ed., Springer-Verlag, Berlin, pp. 269–302.
- [21] Theckes, B., Bouillon, X., and de Langre, E., 2011, "Damping by Branching: A Bionspiration From Trees," *Bionspiration Biomech.*, **6**(4), p. 046010.