Fingering in granular flows

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The flow of granular materials on inclined planes is of interest within the contexts of both industrial processing of powders and geophysical instabilities such as landslides and avalanches[1,2]. These flows have been found to be complex, exhibiting several different flow regimes[3-7] as well as particle segregation effects[8,9] and instabilities[10]. Here we describe an instability that occurs when a front of granular material propagates down a rough inclined plane. The front, which is initially uniform in cross-section, rapidly breaks up into fingers. Although this is similar in appearance to the instability seen in viscous fluids flowing down a plane[11-17], in these latter cases the instability is driven by surface tension, whereas granular materials have no surface tension. We show that the fingering instability in this case is instead induced by the size segregation that develops during the flow.

The experimental set-up is shown in Fig. 1. A 2-m-long and 70-cm-wide glass surface is roughened by gluing onto its surface a single layer of 0.5-mm-diameter glass beads. The rough bed is transparent and allows the system to be viewed from below. A double gate system (Fig. 1a) can be opened suddenly to give an opening h_g constant across the bed. When the inclination angle α is sufficiently large, the material pours through the gate, forming a well defined front propagating down the slope.

The preliminary experiments were carried out using poor-quality glass beads 0.5 mm in mean diameter (material no. 1). By 'poor quality' we mean that the glass beads were not properly sieved and contained some clusters of beads bonded together forming large and irregular particles. With this material the propagation of the front exhibits an instability as shown in Fig. 1b: the front rapidly breaks up into fingers. This instability is observed for a wide range of inclination angle α between 26° and 34° and for different gate openings h_g to a maximum of 16 particle diameters. For higher inclinations or deeper layers, the flow is continuously accelerating or leads to other free-surface instabilities.

A first step in the understanding of the origin of the fingering was obtained from experiments carried out with a second set of beads (material no. 2) which were quasi-monodispersed spherical glass beads sieved between 0.45 and 0.56 mm. With this medium no fingering was observed, and the front remained uniform, regardless of the inclination or the gate opening.

This surprising result indicates that the polydispersity of the granular medium plays an important role in the instability. More precisely, we have found that a necessary condition for the occurrence of the fingering is the presence of coarse irregular particles in the material. The addition to the spherical glass beads of 5% by volume of larger irregular-shaped particles (crushed fruit stones, material no. 3) sieved between 0.50 and 0.63 mm is sufficient to induce the development of fingers. In contrast, the addition of small irregular particles sieved under 0.40 mm leads to a stable front. The fingering is thus entirely driven by the motion of the coarse irregular particles present in the material, suggesting that the segregation occurring during the flow plays a crucial role.

It is well known that, in inclined chute flows of polydispersed media, the coarse particles come to the free surface[7-9]. This phenomenon can be explained by a statistical sieving mechanism[8,9]. In the case of propagating fronts, the vertical segregation occurring far from the front gives rise to a complex recirculation zone [7,10,18] at the front; in our experiments, this recirculation turns out to be the origin of the fingering instability.

We have studied this recirculation zone for materials made of a mixture of the spherical glass beads (material no. 2) and the crushed

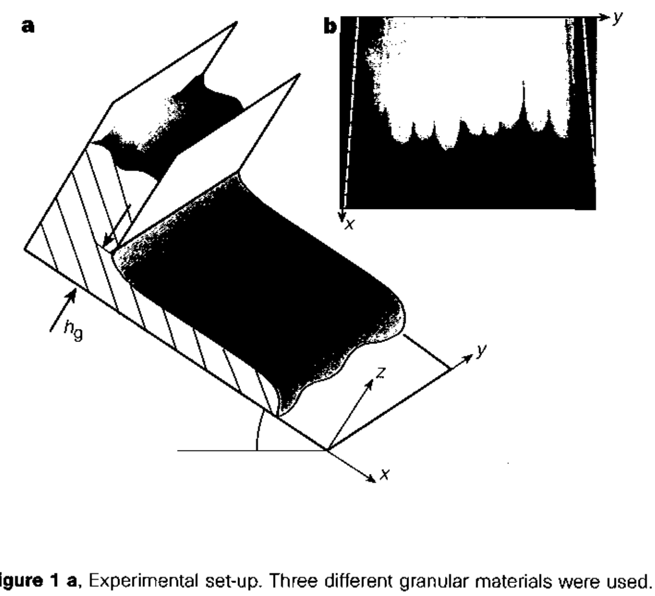

Figure 1 a, Experimental set-up. Three different granular materials were used. Material no. 1: poor-quality glass beads, mean diameter $d = 0.5$ mm; the mean density in close packing was $\rho = 1.5\,\mathrm{g\,cm^{-3}}$ and the bed friction angle was $\delta = 26°$ (δ here is the critical angle at which a 10-diameter-thick layer flows). Material no. 2: quasi-monodispersed spherical glass beads $0.45 < d < 0.56$ mm, $\rho = 1.6\,\mathrm{g\,cm^{-3}}$, $\delta = 22°$. Material no. 3: vegetal abrasive (crushed fruit stones) $0.5 < d < 0.63$ mm, $\rho = 0.7\,\mathrm{g\,cm^{-3}}$, $\delta = 33°$. **b**, Deformation of the front observed for the poor-quality glass beads (material no. 1), $\alpha = 30.5°$, $h_g = 6$ mm.

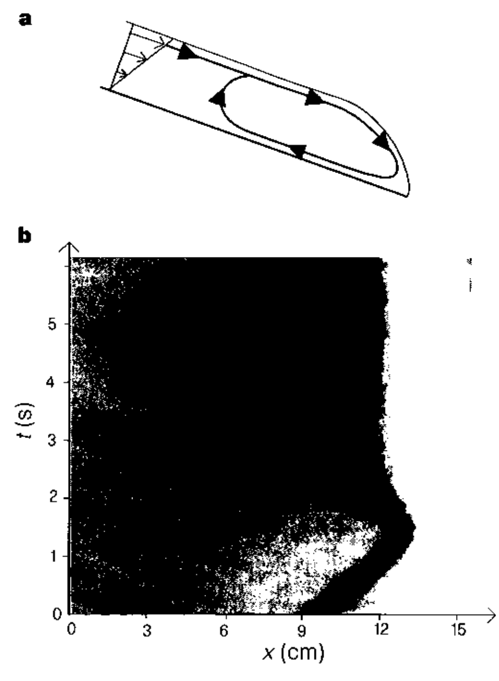

Figure 2 a, Sketch of the recirculation of the large particles at the front. **b**, Spatio-temporal evolution of a line of black tracers (material no. 3) released on a flow of glass beads (material no. 2); $\alpha = 24°$, $h_g = 4$ mm.

fruit stones (material no. 3). The motion of the large particles that we observed in our experiments is sketched in Fig. 2a. At the outlet of the reservoir the large particles rapidly segregate and arrive at the front flowing on the free surface where the velocity is higher. They reach the front and stop on the bed, while the front continues to propagate down the slope. The large particles are thus re-injected in the material (Fig. 2a). The particles then rise up again to the free surface as the segregation process takes place, giving rise to a recirculation motion in a frame moving with the front. Experimental evidence of this phenomena is given in Fig. 2b which represents the spatiotemporal evolution of a line of large black particles of material no. 3 released at the free surface of a flow of pure glass beads. Pictures were taken by a CCD (charge-coupled device) camera placed below the bed and moving with the mean front velocity. The glass beads being translucent, the grey level on the pictures is approximately proportional to the mean concentration of tracers integrated across the avalanching layer. Figure 2b thus gives the time evolution of the tracers' concentration profile close to the front. The black tracers are released 4 cm upstream from the front at $t = 0$ s. They reach the front for the first time at $t = 1.3$ s and then move backward (as sketched in Fig. 2a) before rising up to the free surface; this gives the V-shape on the spatiotemporal diagram. The next loop followed by the tracers is more difficult to observe: the recirculation length appears to vary considerably from one particle to another (over the range 1–12 cm) and also for a given particle from one loop to another, leading to a rapid enlargement of the tracer strip. However, the large particles stay in the vicinity of the front as shown by the enhancement of the grey level close to the front in Fig. 2b for $t > 3$ s. The details of this recirculation process will not be examined here. Rather, we focus on the mechanism of the fingering instability resulting from this basic motion of the large irregular particles.

The initiation of the instability is sketched in Fig. 3a and is entirely driven by the path of the large particles which arrive at the front line on the free surface (black arrows) and leave it at the bed (white arrows). If a small perturbation occurs at the front, the trajectories of the large particles arriving at the front are deflected towards the dip of the deformation, following the steepest slope of the free surface. This is clearly observed in Fig. 3b which is the average of 28 pictures taking every 0.02 s. However, the return trajectories of the coarse particles when they have just left the front remain approximately straight lines: they are close to the bed where the velocity in the laboratory frame is zero (this picture is correct only close to the front where the large particles have not yet moved up to the free surface). A uniform concentration of large particles arriving at the free surface thus leads to a non-uniform distribution at the bed with high concentration at the dip of the deformed front (Fig. 3a and c). This local increase of the concentration of large particles, together with the fact that they are irregular particles having a larger coefficient of friction, leads to a local increase of the friction. The material thus locally slows down which promotes amplification of the deformation which ultimately leads to the formation of fingers. The recirculation motion of the large particles amplifies this phenomenon. The accumulation of the large particles in lines resulting from the mechanism proposed in Fig. 3a is observed at the beginning of front deformation, as shown in Fig. 3c.

This distribution in strips perpendicular to the mean motion is reminiscent of the axial segregation observed in rotating drums (ref. 19 and references therein). However, the basic flows in both configurations are so different that no *a priori* analogy could be suggested. The fingering observed here however, could be related to the pattern formation observed in a rotating drum by Caponeri *et al.*[20]. The connection between those two phenomena and the role of the fingering for the mixing properties in rotating drums represent interesting topics for future investigation. Another interesting topic in a geophysical context would be to study whether this segregation-induced fingering mechanism is relevant for real landslides, which sometimes exhibit finger-like patterns. □

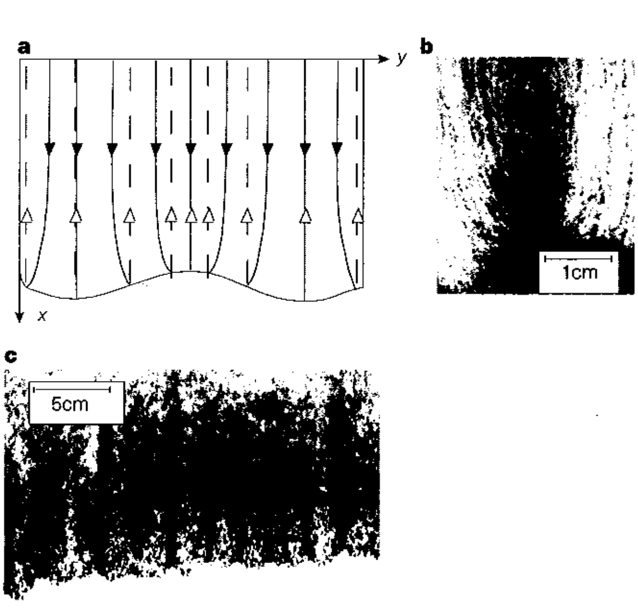

Figure 3 a, Instability mechanism; the black (white) arrows represent the trajectories of the coarse particles on the top (bottom) of the avalanching material. **b**, Averaged picture of 28 top views of the front taken every 0.02 s; **c**, bottom view of the front. The mixture is comprised of 5 vol.% material no. 3 and 95 vol.% material no. 2; $\alpha = 25°$, $h_g = 5$ mm.

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