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Shapes of hanging viscous filaments

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Abstract – We discuss the evolution of the shape of viscous filaments (such as honey threads) placed horizontally in the gravitational field. When attaching both ends of the filament to solid walls, the center of the filament falls down. Hence, a catenary forms and extends as a function of time, owing to gravity. However, it was noted by Koulakis *et al.* that a second shape roughly consisting of three perpendicular pieces (evoking a flying trapeze) is sometimes observed. We try here to understand the origin of this U-shape. We show in particular that its origin is independent of the liquid viscosity, and fixed only by the geometric characteristics (length and radius) of the (initial) filament.

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Viscous cylindrical filaments belong to everyday life: each time we pour a liquid, viscous jets form and fall down, and the characteristics of such jets have been the object of many investigations [1]. Very naturally, the dynamics of the Plateau-Rayleigh instability was described [2], as well as the remarkable behaviors taking place when the jet hits a solid (coiling) [3,4], or a bath of the same oil (air entrainment) [5]. Here we discuss what happens when attaching both ends of a viscous filament to solid walls, so that the filament falls in the gravity field, yet remaining stuck by its ends. As a result, a (kind of) catenary shape quickly shows up, and gets stretched as time goes on. Teichman and Mahadevan comprehensively described the dynamics of this viscous catenary [6], and scaling arguments were proposed by Brochard-Wyart and de Gennes to capture the fall dynamics [7]. Experiments performed by Koulakis *et al.* were found to agree very convincingly with the predicted laws, for both the shape of the filament and its dynamics [8]. However, the same authors also observed that "thin" filaments may rather adopt a kind of U-shape, with two (roughly) vertical parts connected by a horizontal thread. Our aim here is to understand the origin of this second shape and to describe its main characteristics.

The experiment consists of placing a drop of viscous liquid (silicone oils with a viscosity η between 1 Pa s and 100 Pa s, or honey with $\eta = 5$ Pa s) between two parallel solid walls, one of them being mobile. Owing to a quick lateral motion (the wall is screwed on a horizontal rod which can be moved until it reaches a prescribed distance), a filament of centimetric length L and millimetric diameter D is created. This thread meets each wall with a meniscus, which remains immobile during the experiments because of the large viscosity of the liquids. The length L is defined beyond menisci whose millimetre-size extension is anyway always smaller than L. The filament is filmed with a high-speed camera (Phantom V9), using backlighting in order to improve contrast. Depending on the liquid viscosity, the time scale for the thread's evolution is typically in the range 1–10 s, much larger than the time needed for forming it (less than 100 ms).

We report in fig. 1 two series of images showing the evolution of such viscous threads as a function of time. The thread is made of silicone oil ($\eta = 10$ Pa s) drawn so as to make a filament of (initial) length $L = 25 \pm 1$ mm. In fig. 1(a), the diameter D at the middle of the thread is 1.75 ± 0.05 mm, thicker than in fig. 1(b) ($D = 0.33 \pm 0.05$ mm), where the thread is made from a much smaller volume of liquid.

As observed by Koulakis *et al.* [8], the filament evolution depends on the filament diameter. A thick filament generates a family of catenaries that elongate under the action of gravity, but the diameter along each successive catenary is roughly homogeneous (fig. 1(a)). Conversely, a thin thread quickly takes a U-shape, where it is visible that most of the liquid mass gets transferred towards the

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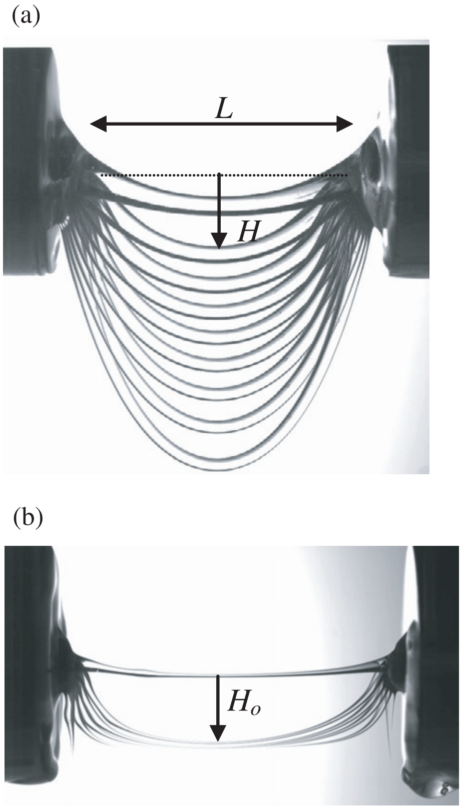

Fig. 1: Evolution of viscous threads hanging between two walls. The threads are obtained by squeezing a drop of silicone oil (viscosity $\eta = 10\,\text{Pa s}$) between the plates, which then are separated by a centimetric distance. (a) The initial thread length L and diameter D are 25 mm and 1.75 mm, respectively, and the interval Δt between two snapshots is 0.08 s. (b) $L = 25\,\text{mm}$ and $D = 0.33\,\text{mm}$; $\Delta t = 0.32\,\text{s}$. The thinnest thread adopts a U-shape, which basically stops at a small depth H_0, contrasting with the catenary of the first series. It eventually breaks, which leads to two hanging disconnected pieces close to the walls.

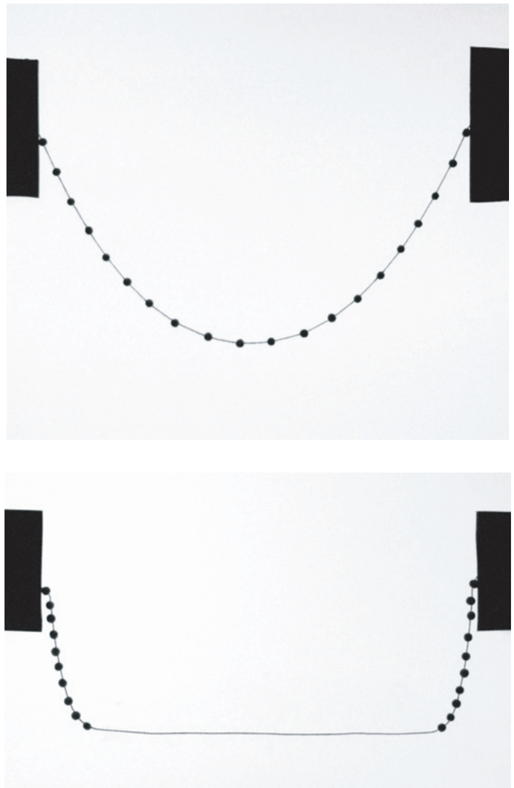

Fig. 2: Shape of a soft solid thread with either a homogeneous distribution of mass (provided by a regular spacing of 10 mm between the millimetric plumbs fixed on it), or an non-homogeneous one; in the second photo, the same number of plumbs is localised close to the fiber ends (distance between plumbs of 5 mm). The distance between the attachment points and the total length of the thread are 13 cm and 20 cm, respectively.

filament ends (fig. 1(b)). As a consequence of this thinning, U-shaped filaments are observed to break systematically before descending very low, contrasting with catenaries, which fall down to depths H much larger than L. The time τ_b after which a filament breaks is typically one second in most of our experiments.

This shape difference can be exemplified by looking at the conformation of a sewing thread, on which fishing plumbs are fixed, either regularly, or concentrated close to the ends of the line, which is attached to solid walls like the viscous threads. It is visible in fig. 2 that the two mass distributions generate very different shapes, similarly to what can be observed in fig. 1: either a catenary for a homogeneous mass, or a U-shape for the heterogeneous one. Looking at these shapes upside down provides the profile of arches, and Gaudí similarly used various irregular distributions of mass along ropes, in order to design the openings in some of his architectures [9–11]!

Very generally, there is no ambiguity about the kind of shape (either catenary or U) selected by the viscous filaments, in particular because of the long horizontal segment that only forms for the U-filament (fig. 1). It is also possible to imagine other tests to distinguish both shapes, as shown in fig. 3. After analyzing numerically the images, we measure here the area A (marked with series of lines in the figure) below the filament and compare it to the surface area HL it occupies in space, as a function of time. Evolutions are found to be different according to the thread shape. On the one hand, the ratio A/HL is roughly constant for a catenary (empty symbols), of approximately 0.3. For a parabolic filament, we indeed expect a constant ratio A/HL, equal to $1/3$, which also holds for a "young" catenary ($H \ll L$) whose shape is close to a parabola. A more developed catenary ($H \sim L$) still provides A/HL of approximately 0.3. On the other hand, the U-shape (full symbols) implies a different behavior: the ratio A/HL keeps on decreasing as a function of time, by a factor higher than 2. For two vertical arms connected by a horizontal segment, A should indeed vanish, but we never reach this limit because the filament eventually breaks at the time τ_b.

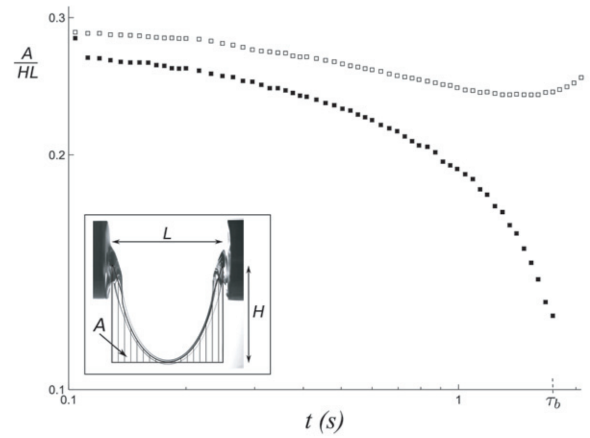

Fig. 3: A criterion for distinguishing catenaries from U-shapes: we calculate the surface area A below the hanging filament (as defined in the insert) and compare it at different times to the total surface area HL of the region occupied by the filament. The data are obtained with threads of viscous silicone oil ($\eta = 10\,\mathrm{Pa\,s}$) of initial length $L = 25\,\mathrm{mm}$. The diameter is either $D = 0.57\,\mathrm{mm}$ (empty symbols) or $D = 0.33\,\mathrm{mm}$ (full symbols). For the thick filament, the ratio A/HL remains nearly constant, close to 0.3 (1/3 would be the value for a parabola or for a catenary of small depth); conversely, the U-shape is characterized by a decrease of A/HL, which would tend towards 0 if the sides became straight. But the filament breaks before, at a time denoted as τ_b.

We now discuss why a shape is selected by a given filament. Assuming that viscosity is the main force opposing the gravitational descent of the liquid as proposed in [6–8], we can first use scaling laws for deriving the dynamics of the catenary, in the spirit of Brochard-Wyart and de Gennes [7]. Along the vertical axis, the gravitational force is $\rho\,\Omega g$, where $\Omega \sim LD^2$ is the volume of the filament. The viscous resistance arises from the existence of axial gradients (along the flow), since the free surfaces induce a plug flow in the thread. Denoting s as the curvilinear length along the filament, the viscous stress can be written dimensionally as $\eta(\mathrm{d}s/\mathrm{d}t)/s$, which must be integrated over the transverse surface area D^2 of the liquid cylinder. Once projected on the vertical direction, this force scales as $\eta(\mathrm{d}s/\mathrm{d}t)/sD^2(H/L) \approx (\eta D^2/L^3)H^2(\mathrm{d}H/\mathrm{d}t)$ (for $H < L$, we have $s^2 \approx 4H^2 + L^2$, from which we deduce the former identity). Balancing it with gravity, we find a non-linear dependence for the kinetics of fall $H^3 \sim \rho g L^4 t/\eta$, in agreement with both experiments [8] and previous models [6–8]. The characteristic time of deformation τ_d for the catenary is obtained for H of order L, which yields:

$$\tau_\mathrm{d} \sim \eta/\rho g L. \qquad (1)$$

For a centimetre-size thread and $\eta = 10\,\mathrm{Pa\,s}$, this time is expected to be around 0.1 s. Capillarity and inertia might also oppose the motion, and it is instructive to compare them with viscosity. A capillary number Ca is built by comparing the viscous force (of the order of $\eta(\mathrm{d}s/\mathrm{d}t)/sD^2$, as seen above) with the capillary force γD (derived below). Ca thus scales as $\eta(\mathrm{d}s/\mathrm{d}t)D/\gamma s$, which yields Ca of order 10 for elongational velocities $\mathrm{d}s/\mathrm{d}t$ of 20 cm/s (see fig. 1(a)), threads of millimetric diameter and centimetric length, and oil of surface tension $\gamma = 20\,\mathrm{mN/m}$ and viscosity $\eta = 10\,\mathrm{Pa\,s}$. On the other hand, denoting ρ as the liquid density, the Reynolds number Re can be defined as $\rho s(\mathrm{d}s/\mathrm{d}t)/\eta$, of the order of 0.1 with the same parameters as above. Hence viscosity dominates both surface tension and inertia, which justifies a dynamics dictated by a balance between gravity and viscosity.

In order to derive eq. (1), it was also assumed that the thread thickness is constant along the curvilinear coordinate. However, surface tension squeezes a liquid cylinder in order to reduce its surface area, which generates an axial flow along it. The surface energy of a cylindrical thread scales as γDL, from which we deduce an axial capillary force γD. The same result can be obtained by integrating over the area D^2 the Laplace pressure γ/D that drives the liquid towards the thread ends. Viscosity resists this motion. Since the velocity gradients are axial, as emphasized earlier, the viscous stress is expected to scale as $\eta V/L$, which yields a force $\eta V D^2/L$. Balancing it with the capillary driving force γD, we deduce a "drainage" velocity V of the order of $(\gamma/\eta)\ (L/D)$. The corresponding drainage time $\tau_\mathrm{c} \sim L/V$ can thus be written as

$$\tau_\mathrm{c} \sim \eta D/\gamma. \qquad (2)$$

For millimetric (thick) viscous jets ($\eta = 10\,\mathrm{Pa\,s}$), τ_c is approximately 1 s, but it can become smaller than the deformation time τ_d given by eq. (1) if the jet becomes either thin (small τ_c) or short (long τ_d). More generally, the comparison between both these times should decide the shape of the thread. In particular, the U-shape should be privileged if $\tau_\mathrm{c} < \tau_\mathrm{d}$, that is:

$$LD < a^2, \qquad (3)$$

where a is the capillary length ($a = (\gamma/\rho g)^{1/2}$), which is 1.5 mm for silicone oil and 2.2 mm for honey. If the criterion (3) is obeyed, capillary drainage is quicker than sagging and the thickness of the jet is no longer homogeneous as it falls. The mass redistribution towards the edges leads to a U-shape, as shown in fig. 1(b). Remarkably, the viscosity, which opposes both gravity and capillary flows, does not enter this criterion, which is found to be purely geometrical: the thread length and/or diameter alone fix the shape.

We tested these ideas by monitoring the shape of viscous threads of various geometries and viscosities (silicone oils with $\eta = 1$, 10 and 100 Pa s, and honey with $\eta = 5\,\mathrm{Pa\,s}$). We report our results in fig. 4, where each data point corresponds to a different thread, and where the symbols are empty or full for catenaries or U-shapes, respectively.

For a fixed diameter ($D/a \approx 0.4$), short threads are U-shaped while long ones form catenaries, as expected

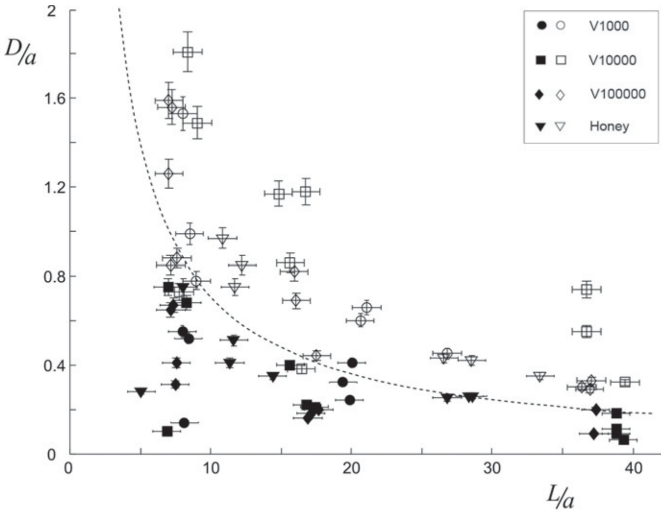

Fig. 4: Phase diagram for the shape of a viscous filament in the field of gravity. We vary the length L and the diameter D of the filament, and normalize them by the capillary length a. Filaments are made of silicone oil of viscosity $\eta = 1\,\mathrm{Pa\,s}$ (circles), $10\,\mathrm{Pa\,s}$ (squares) and $100\,\mathrm{Pa\,s}$ (diamonds), or honey of viscosity $\eta = 5\,\mathrm{Pa\,s}$ (triangles). Each data point corresponds to a filament, and its shape is indicated by a colour: black and white symbols hold for U-shapes and catenaries, respectively. The dotted line is a hyperbola (eq. (3)), of equation $LD = 7a^2$.

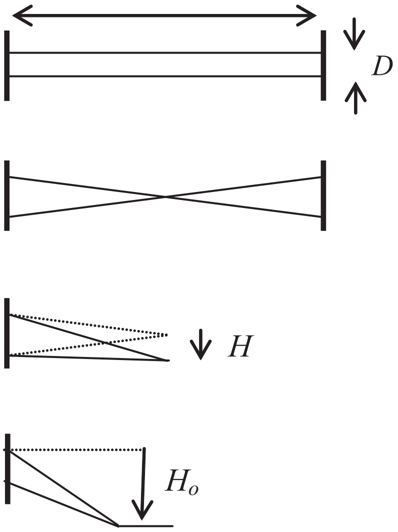

Fig. 5: Evolution of a U-thread of initial length L and diameter D. Capillary drainage first makes the thread profile conical; then the cones (only the left one was represented) fall down, owing to the action of gravity, and stop at a depth H_0 s observed in fig. 1(b), where they eventually break.

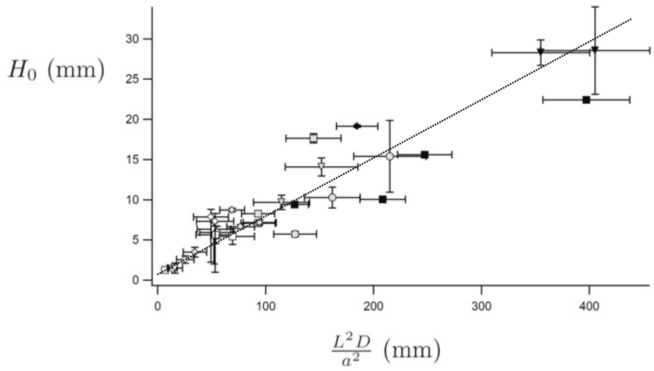

Fig. 6: Final depth H_0 of U-filaments, as a function of their geometry. Filaments are made of silicone oil of viscosity $\eta = 1\,\mathrm{Pa\,s}$ (circles), $10\,\mathrm{Pa\,s}$ (squares) and $100\,\mathrm{Pa\,s}$ (diamonds), or honey of viscosity $\eta = 5\,\mathrm{Pa\,s}$ (triangles). Their initial length is $L \approx 12\,\mathrm{mm}$ (white symbols), $L \approx 25\,\mathrm{mm}$ (grey symbols) and $L \approx 57\,\mathrm{mm}$ (black symbols). The depth is observed to scale fairly well with the geometrical quantity DL^2/a^2, as expected from eq. (4).

from our discussion (eq. (3)). For a fixed length ($L/a \approx 8$, for example), only thin filaments are U-shaped. As predicted by eq. (3), a hyperbola (of equation $D \sim 7a^2/L$, and drawn in fig. 4 with a dotted line) separates convincingly both domains. We can notice that the critical length $L_c \sim a^2/D$ above which gravity effects dominate capillary effects, has the same structure as the height of the capillary rise in a tube of diameter D —which is similarly given by a balance between capillarity and gravity, and also (trivially) independent of viscosity.

U-filaments' drainage is mainly horizontal, which generates a kind of conical shape, as sketched in fig. 5. These cones fall, but not very low: unlike catenaries, the fall stops at a depth H_0, which we now discuss.

The stop of the fall indicates that surface tension, which mainly acts in the direction of the cone, is able to balance gravity. Projecting the surface force on the vertical axis (for $H_0 < L$), this balance can be written, $\rho g L D^2 \sim \gamma D H_0/L$, which immediately yields

$$H_0 \sim L^2 D/a^2. \qquad (4)$$

In the U-regime (eq. (3), $LD < a^2$), H_0 is indeed found to be smaller than L. More quantitatively, we report in fig. 6 the final depth H_0 of U-filaments as a function of the length L^2D/a^2 suggested by eq. (4). The shape of the symbols indicates the nature of the liquids (as defined in the previous figures), and both the initial length and the thread diameter were varied: white symbols correspond to $L \approx 12\,\mathrm{mm}$, grey symbols to $L \approx 25\,\mathrm{mm}$ and black symbols to $L \approx 57\,\mathrm{mm}$. It is observed in fig. 6 that H_0 increases significantly with L, and also with D (which is varied systematically, for a given L). The data are fairly well described by a straight line passing through the origin, in agreement with eq. (4). The slope, of the order of 0.08, remains to be understood.

The same result can be derived from dynamical considerations. As a cone fall on a distance H (fig. 5), its length increases by a quantity HD/L, which implies a shear velocity scaling as $(\mathrm{d}H/\mathrm{d}t)D/L$, and thus a shear rate of $(\mathrm{d}H/\mathrm{d}t)/L$ (because the cone distorts as it falls, the dominant velocity gradients are across the diameter D). The resulting viscous force $[\eta(\mathrm{d}H/\mathrm{d}t)/L]D^2$ is balanced

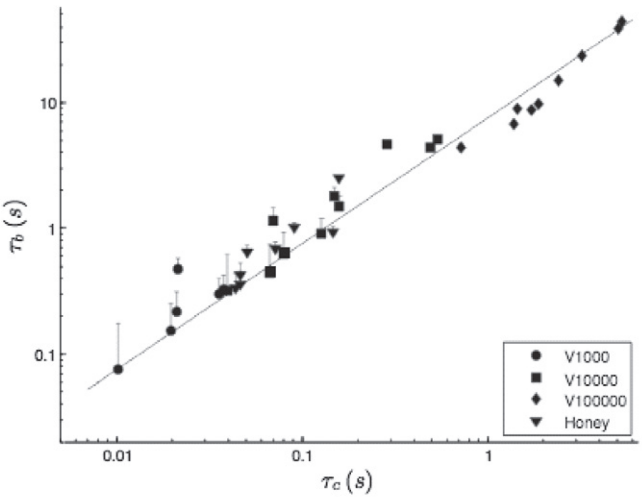

Fig. 7: Lifetime τ_b of viscous U-threads. At $t = \tau_b$, the filament spontaneously breaks, generally close to its middle, as seen in the high-speed movies following the evolution of the thread or in fig. 1(b). This time is plotted as a function of the quantity $\tau_c \sim \eta D/\gamma$, calculated for each thread. Different liquids are used (same symbols as in fig. 4). τ_b is observed to be directly proportional to τ_c, as indicated by the full line, of slope 1.

along the fall by the weight $\rho g L D^2$, which yields a fall velocity $(\mathrm{d}H/\mathrm{d}t)$ scaling as $\rho g L^2/\eta$. This regime is valid only during the time τ_c, after which the liquid is drained. Hence we expect the final depth H_0 to be $\rho g L^2 \tau_c/\eta$, which (using eq. (2)) brings us back to eq. (4).

We can finally check that the extensional dissipation (arising from axial gradients $(\mathrm{d}L/\mathrm{d}t)/L$) remains smaller than the shear $(\mathrm{d}H/\mathrm{d}t)/L$ in the cylinder considered here: since we have $(\mathrm{d}L/\mathrm{d}t)L \sim (\mathrm{d}H/\mathrm{d}t)H$, the quantity $(\mathrm{d}L/\mathrm{d}t)/L$ is $(\mathrm{d}H/\mathrm{d}t)/L$ times H/L, where the ratio H/L indeed remains smaller than unity in the U-regime $(LD < a^2)$.

As already emphasized, U-filaments thin and eventually pinch off. The break time τ_b can be expected to be a "Rayleigh time", *i.e.* the time necessary for a viscous cylinder to break into droplets owing to the action of surface tension. Hence this time should be of the order of the capillary time evaluated in eq. (2). In fig. 7, we compare the measured τ_b with the calculated capillary time $\tau_c \sim \eta D/\gamma$.

The data are plotted in a log-log representation, in order to make the variety of times explored clear (more than two orders of magnitude). Whatever the liquid viscosity, the data collapse on a straight line of slope 1 drawn with a full line: the lifetime τ_b of the thread is found to be approximately $7\tau_c$.

We could finally wonder whether transitions between both shapes are possible as time goes by. We never observed such transitions, which can be simply understood. On the one hand, for U-filaments obeying criterion (3), the thread diameter keeps on decreasing while the total length is roughly constant (and close to L), so that LD remains smaller than a^2. On the other hand, a catenary $(LD > a^2)$ gets thinner as time goes by, as observed in fig. 1(a), so that the quantity $LD \sim \Omega/D$ keeps on increasing: the filament can conserve its shape all along the fall. This also allows us to understand why a (purely geometrical) static criterion can fix the shape, which however sets dynamically.

It would be interesting to see how these ideas hold, or not, for non-Newtonian fluids [12]. Since the viscous resistance arises from elongational flows, extensional viscosity is the fluid parameter that sets the dynamics of the processes. For Newtonian fluids, it is just three times the dynamic viscosity, which thus does not affect any of our scaling laws. In contrast, extensional viscosity can become extremely high for solutions of polymer or concentrated soap, which should impact the shape and/or the dynamics of the thread. The study of the thread shape in this case is under progress.

* * *

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