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Vortex shedding modeling using diffusive van der Pol oscillators

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Abstract A simple model for the near wake dynamics of slender bluff bodies in cross-flow is analyzed. It is based on a continuous distribution of van der Pol oscillators arranged along the spanwise extent of the structure and interacting by diffusion. Diffusive interaction is shown to be able to model cellular vortex shedding in shear flow, the cell size being estimated analytically with respect to the model parameters. Moreover, diffusive interaction succeeds in describing qualitatively the global suppression of vortex shedding from a sinuous structure in uniform flow. ***To cite this article: M.L. Facchinetti et al., C. R. Mecanique 330 (2002) 451–456.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

fluid mechanics / vortex shedding / 3-D spanwise effects / dynamical systems / van der Pol oscillator

Modélisation du détachement tourbillonnaire avec des oscillateurs de van der Pol interagissant par diffusion

Résumé Nous analysons un modèle simple de la dynamique du sillage proche derrière une structure élancée. Le modèle est constitué par une distribution continue, le long de la structure, d'oscillateurs de van der Pol interagissant par diffusion. En écoulement cisaillé, la diffusion permet de décrire le détachement tourbillonnaire par cellules, dont la taille est ici calculée analytiquement en fonction des paramètres du modèle. Dans le cas d'une structure sinueuse en écoulement uniforme, le modèle reproduit qualitativement la suppression globale du détachement tourbillonnaire. ***Pour citer cet article : M.L. Facchinetti et al., C. R. Mecanique 330 (2002) 451–456.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

mécanique des fluides / détachement tourbillonnaire / effets 3-D / systèmes dynamiques / oscillateur de van der Pol

Version française abrégée

On considère une structure élancée fixe soumise à un écoulement transverse stationnaire [1]. Le phénomène du détachement tourbillonnaire derrière la structure peut présenter d'importants effets tridimensionnels. C'est le cas du détachement tourbillonnaire par cellules, en écoulement cisaillé [2–4] : en présence d'une distribution régulière de la vitesse de l'écoulement amont, le spectre du sillage proche montre une variation par palier, sous forme de cellules caractérisées par une fréquence constante et

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délimitées par des discontinuités localisées. Un deuxième effet concerne la suppression du détachement tourbillonnaire en écoulement uniforme derrière une structure ayant un profil sinueux dans la direction du courant [5].

Ces phénomènes ont déjà été modélisés par simulation numérique directe des équations de Navier–Stokes, mais avec des limites importantes concernant le nombre de Reynolds et l'élancement de la structure. Des modèles dynamiques à petit nombre de degrés de liberté, notamment des réseaux d'endomorphismes du cercle [6], l'équation de Ginzburg–Landau complexe [7] et des réseaux d'oscillateurs de van der Pol couplés [8–10], représentent donc un outil d'analyse plus simple pour la compréhension de la physique de ces phénomènes.

On considère ici une distribution continue d'oscillateurs de van der Pol en interaction par diffusion le long de la structure (1). Ces oscillateurs modélisent le caractère fluctuant du détachement tourbillonnaire dans le sillage proche à travers la seule variable réelle $q(z,t)$. Dans cette Note on analyse les propriétés dynamiques d'un tel système d'un point de vue analytique.

Dans la deuxième section, le cas d'une structure stationnaire élancée en courant cisaillé est abordé (Fig. 1(a)). Le comportement dynamique du modèle est analysé en considérant des solutions sous la forme d'ondes propagatives, pour lesquelles on obtient, approchée au premier ordre en fréquence, une relation de dispersion non-linéaire (2). En cherchant des ondes neutres stables, on déduit la relation (3) entre l'amplitude Q des oscillations et le nombre d'onde k. Cette relation permet d'expliquer l'apparition des défauts de phase (dislocations) dans l'évolution spatio-temporelle du système dynamique (Fig. 2). En écoulement cisaillé, le gradient de phase, ou nombre d'onde k, est une fonction croissante du temps (4) et l'amplitude des ondes qui en résulte décroît vers zéro. Dans cette limite, une discontinuité est attendue : l'annulation de l'amplitude permet une indétermination de la phase, donc une dislocation peut se manifester [11]. Une longueur d'onde caractéristique λ_c apparaît (5). Le passage de cette analyse locale aux effets d'ensemble sur le contenu spectral des oscillations (Fig. 3) permet de relier λ_c à la taille des cellules du spectre. La comparaison avec des simulations numériques du modèle (1) et des données de la littérature [10] valide cette approche (Fig. 4).

Dans la troisième section, le cas d'une structure stationnaire élancée avec une géométrie sinueuse dans la direction de l'écoulement (uniforme) est abordé (Fig. 1(b)). Le modèle des oscillateurs de van der Pol (1) est réécrit afin de prendre en compte le temps de retard par convection qui intervient dans l'interaction spatiale (6), dans le même esprit que [12] pour des oscillateurs de Landau. Le comportement dynamique du modèle est analysé en considérant des solutions sous la forme d'ondes stationnaires, pour lesquelles on obtient, approchée au premier ordre en fréquence, une relation analytique entre l'amplitude Q et les paramètres géométriques du profil sinueux. Ceci est en accord qualitatif avec les observations expérimentales et numériques de la littérature [5]. Une augmentation de l'amplitude X_0 et du nombre d'onde k du profil sinueux provoque la suppression progressive et globale du détachement tourbillonnaire.

Deux effets tridimensionnels du détachement tourbillonnaire derrière une structure stationnaire élancée soumise à un écoulement transverse stationnaire sont donc ici modélisés, qualitativement et quantitativement, par un système dynamique à petit nombre de degrés de liberté. Une distribution continue, le long de la structure, d'oscillateurs de van der Pol en interaction par diffusion constitue un outil d'analyse efficace pour la compréhension de la physique des deux phénomènes.

1. Introduction

Since Strouhal's analysis of æolian tones, the modeling of vortex shedding behind a stationary bluff body has been widely analyzed [1]. Three-dimensional (3-D) phenomena in uniform and sheared flow are of increasing interest, both for fundamental research in non-linear dynamics and applied offshore engineering.

Cellular vortex shedding is the most distinctive phenomenon observed in the near wake of a uniform bluff body under shear flow (Fig. 1(a)). The shedding frequency varies discontinuously in the spanwise direction

[2] and this equivalently appears behind a tapered cylinder in uniform flow [3,4]. Another feature of 3-D vortex shedding is its suppression by means of mild geometric disturbances, such as spanwise waviness (Fig. 1(b)) [5]. To investigate the underlying physics, several low-order models have been proposed in the literature, such as non-linear circle map oscillators [6], complex Ginzburg–Landau equations [7], and continuous distributions of van der Pol oscillators [8–10].

The present Note explores the dynamical properties of continuously distributed and diffusive interacting van der Pol oscillators from an analytical point of view. Some of these properties have already been identified by other authors using numerical simulations, but never isolated in a closed form. In particular, cellular vortex shedding under shear flow and vortex shedding suppression from a sinuous structure are analyzed.

2. Vortex shedding from a straight structure in shear flow

Let us consider a stationary slender circular cylinder of diameter D, subject to shear cross-flow of free stream velocity $U(Z) = U_0(1 + \beta Z)$, β being a shear parameter (Fig. 1(a)). Following Strouhal's law [1], the local vortex shedding is expected to occur at the angular frequency $\omega_0(1 + \beta Z)$, where $\omega_0 = 2\pi S U_0/D$ and S is the Strouhal number. However, it appears [2] that the shedding frequency varies step-by-step along the structure, forming cells of constant frequency (Fig. 1(a)).

The fluctuating nature of the near wake vortex street behind the structure is now modeled by a continuous distribution of van der Pol oscillators, arranged along the extent of the structure and interacting by diffusion. Scaling the time and space variables with respect to $1/\omega_0$ and D respectively, the model can be written [9,10]

$$\frac{\partial^2 q}{\partial t^2} + \varepsilon \Omega(z)\left(q^2 - 1\right)\frac{\partial q}{\partial t} + \Omega^2(z) q - \nu \frac{\partial^3 q}{\partial t \partial z^2} = 0 \tag{1}$$

where $q(z,t)$ is a dimensionless variable describing the near wake flow, ε is the positive parameter of the van der Pol oscillator, ν is a diffusion parameter, and $\Omega(z) = 1 + \beta z$ the dimensionless vortex shedding angular frequency. Note that in model (1) the parameter ε does not affect the finite amplitude $Q = 2$ of the limit cycle, in opposition to the Ginzburg–Landau equation such as in [7], where it expresses the influence of the Reynolds number and thereby the distance to the threshold of the Hopf bifurcation at the onset of the Bénard–von Karman vortex street. A traveling wave solution is sought in the form $q(z,t) = Q\,\mathrm{e}^{\mathrm{i}(kz-\omega t)}$. A scale separation in space is assumed requiring $\beta \ll k$ and the first harmonic approximation in time is

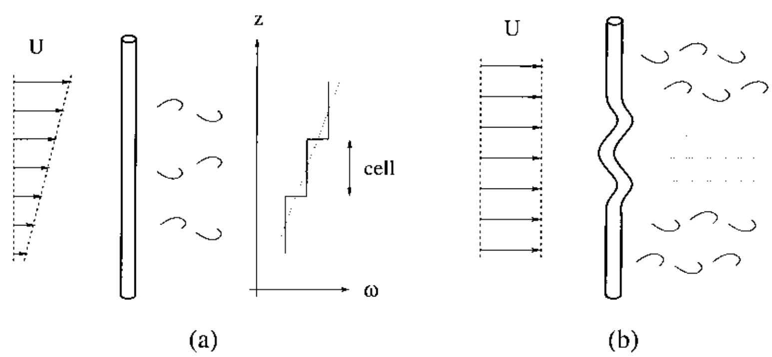


Figure 1. (a) Cells of vortex shedding frequency in shear flow. (b) Suppression of vortex shedding from a sinuous structure.

Figure 1. (a) *Cellules de détachement tourbillonnaire en écoulement cisaillé.* (b) *Suppression du détachement tourbillonnaire par une structure sinueuse.*

applied. The non-linear dispersion relation associated to equation (1) then becomes

$$\mathcal{D}(\omega, k) = -\omega^2 + \mathrm{i}\left(1 - \frac{Q^2}{4} - \frac{\nu}{\varepsilon}k^2\right)\varepsilon\omega + \Omega^2 = 0 \tag{2}$$

Looking at real wavenumber k and real angular frequency ω means to equate to zero both real and imaginary parts of (2). This implies that $\omega = \Omega$ and

$$Q^2 = 4\left(1 - \frac{\nu}{\varepsilon}k^2\right) \tag{3}$$

Diffusion is then seen to decrease the amplitude of the vortex shedding parameter. The spatio-temporal evolution of the dynamical system (1) may also be represented using a spatial generalization of the local phase portrait, with lines of constant phase $\phi(z,t) = \Omega t$ in the (z,t) plane (Fig. 2). Setting $\phi(z, t = 0) = 0$ as a particular initial condition, iso-phase lines are progressively tilted in time because of the spanwise frequency distribution imposed by the shear. As a temporal harmonic function linearly shifted in space, the fluid variable $q(z,t)$ is then a periodic function of z and a time-varying spanwise phase gradient, or spanwise wavenumber, may be defined as

$$k = \frac{\partial \phi}{\partial z} = \frac{\partial(\Omega t)}{\partial z} = \beta t \tag{4}$$

The spanwise wavenumber k is a growing function of time in (4), so that the amplitude Q given by (3) is progressively reduced down to zero. At this limit a discontinuity is expected: zero amplitude means phase indetermination, which allows for a phase jump and an associated topological defect called a phase dislocation (Fig. 2) [11]. This instantaneously reduces k and allows the oscillation to be set again with a positive amplitude. The mechanism goes on working indefinitely: k increases, so that Q vanishes, then a new dislocation appears and so on. The system is then characterized by a finite critical maximum spanwise wavenumber, corresponding to vanishing amplitude in Eq. (3), $k_c = \sqrt{\varepsilon/\nu}$, so that the critical wavelength becomes

$$\lambda_c = 2\pi\sqrt{\frac{\nu}{\varepsilon}} \tag{5}$$

All this is expected to take place locally in time and space, i.e. at discrete spanwise positions and periodically, as shown by a numerical integration of Eq. (1) (Fig. 3(a)). Computations are performed here applying centered finite difference in space and time, with a second order accurate explicit scheme and vanishing spatial second derivative as boundary condition. The spatio-temporal evolution in Fig. 3(a) preserves the reflectional invariance $z \mapsto -z$ of the basic diffusive model, which is actually broken by the imposed frequency gradient. This supports the effectiveness of the described local behavior in controlling the global dynamics. The local angular frequency is apparently not modified by this process. Nevertheless, on a long time scale, such a regular dislocation pattern yields a cellular spanwise distribution of the power spectral density in time of $q(z,t)$ (Fig. 3(b)), so that spanwise frequency cells of wavelength λ_c naturally

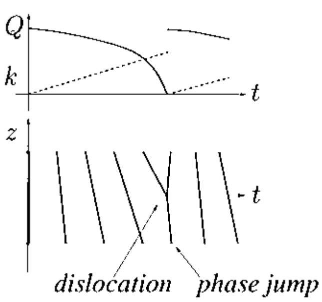


Figure 2. Dislocation occurrence mechanism: phase jumps at vanishing amplitude.

Figure 2. *Schéma d'apparition des dislocations : saut de phase où l'amplitude s'annule.*

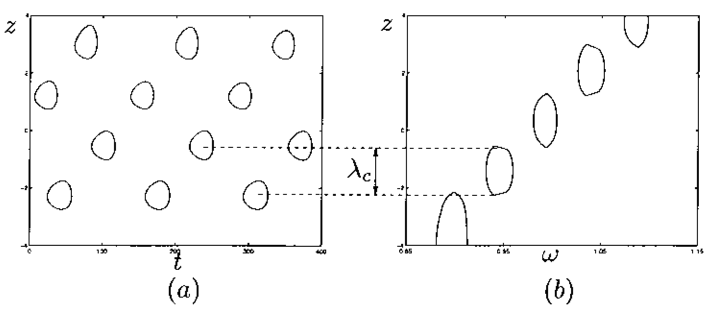
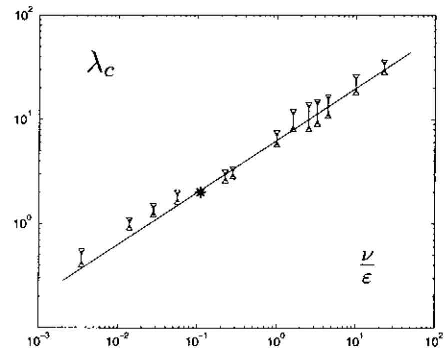


Figure 3. Numerical integration of Eq. (1): (a) dislocation pattern on the amplitude (iso-line of $Q(z,t) = 1$) and (b) time power spectral density (PSD) levels (iso-lines of $PSD(q) = 0.2$). Simulation parameters as in [10]: $\varepsilon = 0.2$, $\nu = 0.022$, $\beta = 0.027$.

Figure 3. *Intégration numérique de l'équation* (1) : (a) *distribution des dislocations en amplitude* (*iso-lignes* $Q(z,t) = 1$) *et* (b) *distribution spatiale de la densité de puissance spectrale* (*iso-lignes* $DSP(q) = 0{,}2$). *Paramètres de la simulation d'après* [10] : $\varepsilon = 0{,}2$, $\nu = 0{,}022$, $\beta = 0{,}027$.

Figure 4. Cell size: '—' Eq. (5); '|' numerical integrations of (1); '*' [10].

Figure 4. *Taille des cellules* : « — » *équation* (5) ; « | » *integrations numériques de* (1) ; « * » [10].



appear. Their spectral energy contents is locked onto their centroid angular frequency, leading to a spanwise step-function organization.

The cell length derived by the previous phase analysis, Eq. (5), is then compared with results of sets of numerical integrations for several values of ν/ε and with the numerical results of Balasubramanian et al. [10] based on experimental evidence (Fig. 4). If the spatial scale separation $1/\beta \gg \lambda_c$ is satisfied, Eq. (5) is a good approximation of the cell length over several order of magnitude of ν/ε. Error bars in Fig. 4 are due to several choices of β. The analyzed parameter range includes numerical values of ν/ε used in [9,10]. These results may not strictly be compared with the related Ginzburg–Landau model, where diffusion is always mixed with stiffness, see [7].

3. Vortex shedding from a sinuous structure in uniform flow

Let us now consider the case of uniform flow on a structure with a spanwise mild sinuous geometric disturbance $X = X_0 \sin(kz)$ (Fig. 1(b)). Spanwise interactions for the near wake variable $q(z,t)$ must take into account the time lag associated to convection behind the sinuous distribution $\tau = \omega_0 X D/U = 2\pi S X_0 \sin(kz)$, such as for Landau oscillators in [12]. Upon introducing $T = t - \tau$, Eq. (1) now reads

$$\frac{\partial^2 q}{\partial T^2} + \varepsilon\left(q^2 - 1\right)\frac{\partial q}{\partial T} + q + \nu\left[-\frac{\partial^3 q}{\partial T \partial z^2} + 2\left(\frac{\partial \tau}{\partial z}\right)\frac{\partial^3 q}{\partial T^2 \partial z} - \left(\frac{\partial \tau}{\partial z}\right)^2 \frac{\partial^3 q}{\partial T^3}\right] = 0 \qquad (6)$$

A standing wave solution whose wavenumber is set by the geometric disturbance,

$$q(z,T) = Q \sin(kz)\cos(\omega T)$$

requires, applying the first harmonic approximation in time and space, that the frequency is unchanged, $\omega = 1$, whereas the amplitude can be written

$$Q^2 = 4\left[1 - \frac{\nu}{\varepsilon}\left(1 + \pi^2 S^2 X_0^2\right)k^2\right] \tag{7}$$

The geometric disturbance decreases the global amplitude through the combined parameter $X_0 k$, as observed in [5]. Suppression is then achieved for values of ν/ε consistent with previous models of van der Pol oscillators [9,10].

4. Conclusion

Some three-dimensional features of vortex shedding in the near wake of stationary slender bluff bodies in stationary flow are qualitatively and quantitatively described by a low-order dynamical model, formed by van der Pol oscillators continuously arranged along the spanwise extent of the structure and interacting by diffusion. Diffusive interaction is effective in modeling cellular vortex shedding due to spanwise shear in the oncoming flow. The proposed analytical relation for the resulting cell size (5) is validated successfully by numerical simulations and previous studies in literature [10]. Moreover, diffusive interaction succeeds in describing qualitatively the global vortex shedding suppression from a sinuous structure in uniform flow. Both phenomena have been related to longitudinal streamwise vortices in the near wake [4,5], which concentrate the vorticity distributed upstream in the oncoming shear flow or generated by spanwise varying separation point along the structure. Cellular shedding and vortex shedding suppression involve a mechanism of vorticity diffusion in the near wake: this supports the success of the spanwise viscous interaction in their modeling.

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