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# Vortex-induced vibrations of cylinders bent by the flow

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#### ABSTRACT

Flexible cylinders exposed to transverse flows may undergo transverse vortex-induced vibrations due to the oscillating lift force imposed by their wake. But the drag on such structures often leads to large in-line deflections that may significantly affect the dynamics through the combined effects of the curvature-induced tension, the local inclination of the cylinder, the non-uniformity of the normal flow profile, and the large axial flow component. In this paper, we investigate the consequences of flow-induced bending on the vortexinduced dynamics of slender cantilever cylinders, by means of numerical simulations. We combine a distributed wake oscillator approach to model the dynamics of the wake with Lighthill's large-amplitude elongated body theory to account for the effect of the axial flow in the reactive (added mass) force. The use of such reduced order models facilitates the identification of the physical mechanisms at play, including through the linear analysis of the coupled fluid-structure system. We find that the primary consequence of flow-induced bending is the inhibition of single mode lock-in, replaced by a multi-frequency response of the structure, and the reduction of the vibration amplitude, as a result of the broadening of the wake excitation spectrum and of the localization of the energy transfer due to the variations induced in the normal flow profile. We also find that the curvature-induced tension is of negligible influence, but that the axial flow component may on the other hand significantly alter the dynamics owing to the destabilizing effect of the reactive force on the structural modes.

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#### 1. Introduction

The vortex-induced vibrations (VIVs) of slender cylindrical structures has been a prominent subject of research for many years. Originally, a better understanding of this phenomenon was sought in the civil and marine engineering community mostly because of the damage it may cause on a number of flow-exposed structures such as buildings, power transmission lines, marine risers, towing cables, or mooring lines. For extensive reviews regarding VIVs, the reader is referred to Williamson and Govardhan (2004), Sarpkaya (2004), Williamson and Govardhan (2008), Bearman (2011) and Wu et al. (2012). More recently, a renewed interest for the VIVs has arisen from the potential they bear as an alternative source of energy (Bernitsas et al., 2008).

The large majority of existing studies focus on the VIVs of straight cylinders in a variety of configurations: rigid or flexible, perpendicular to the flow or slanted, exposed to a uniform or a sheared flow. However, most of the off-shore flexible structures such as those cited above are actually greatly deformed in the direction of the flow under the effect of the free-stream. This configuration differs from the case of a straight cylinder on several aspects. First, the deflection in the plane of the free-stream of a cylinder is responsible for a curvature-induced tension inside the structure. The tensioning

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of the cylinder may affect its natural frequencies, and consequently its dynamic response to the wake excitation. Secondly, deflected structures are not locally perpendicular to the flow, which modifies the features of vortex shedding in the wake and the associated forces on the structure. Besides, a curved structure experiences a spanwise variation of its angle with the free-stream. Finally, the reconfiguration of the structure leads to a large axial component of the flow on the most inclined portion of the structure that may even become dominant when the deflection is significant. The consequences of some of these specificities have been individually studied, see for instance Srinil et al. (2009) and Srinil (2010) for the structural effect of the curvature, Lucor and Karniadakis (2003), Facchinetti et al. (2004b), Franzini et al. (2009), Jain and Modarres-Sadeghi (2013) and Bourguet et al. (2015) for the effect of the inclination, or Vandiver (1993), Chaplin et al. (2005), Trim et al. (2005), Lucor et al. (2006), Violette et al. (2010) and Bourguet et al. (2013) for the effect of non-uniform normal flow profiles, but their combined effects may lead to a significant alteration of the wake–structure interaction that has not yet been fully investigated.

As a first step towards the understanding of VIVs of bent cylinders, Miliou et al. (2007) and de Vecchi et al. (2008) numerically explored the vortex shedding process in the wake of a rigid cylinder in the shape of a convex or concave quarter of a ring, when the structure is respectively fixed or forced into an oscillatory motion. Building on these results, Assi et al. (2014) and Seved-Aghazadeh et al. (2015) experimentally investigated the free vibrations of similar structures and found that the amplitude of the oscillations is much reduced compared to the straight configuration. Two studies provided experimental observations regarding the VIVs of flexible structures about a curvy shape: the experimental work of Zhu et al. (2016) considered the vibrations of a naturally concave-shaped cylinder subjected to a shear flow, while that of Morooka and Tsukada (2013) tested a model riser deformed in the shape of a concave catenary under the effect of a uniform free-stream. Finally, Bourguet et al. (2012, 2015) numerically investigated the VIVs of tensioned flexible beams respectively exposed to a normal sheared flow and an inclined uniform flow. Both studies considered the influence of a small average in-line deformation and noted the transition from a mono-frequency to a multi-frequency response associated with a modification of the normal flow profile due to the bending. A reduction of the amplitude of the VIVs was also reported in Bourguet et al. (2015). At this point however, a theoretical study is still missing to clarify the consequences of flow-induced bending on the VIVs of slender cylinders in large deformations and identify the physical mechanisms at play. One may for instance wonder how the bending-induced shear in the normal flow might impact the vibration spectrum, whether lock-in may or may not still occur, or furthermore how the amplitude of vibration might be affected?

But the flow-induced bending of the structure may have even more dramatic consequences. Indeed, slender structures in axial flows are liable to a flutter instability (Datta and Gottenberg, 1975; Yadykin et al., 2001; Païdoussis et al., 2002; Semler et al., 2002). This self-induced dynamics results from the destabilizing effect of the inviscid pressure forces associated with the deformation of the structure in a free-stream with a significant axial component (Eloy et al., 2007; Singh et al., 2012a). When a cylinder deflects in a transverse flow, the increasing spanwise component of the free-stream may thus be the cause of such instability. More generally, the influence of the inviscid pressure forces on the structural modes may have consequences on the vortex-induced dynamics even in a domain of the parameter space where the system does not flutter.

The purpose of the present paper is to provide an analysis of the small-amplitude vibrations of slender cylinders bent by the flow by means of reduced order models to identify the physical mechanisms at play. In particular, a formulation of the inviscid pressure forces based on Lighthill's large-amplitude elongated body theory (Lighthill, 1971) will be used to account for the destabilizing effect of the axial component of the free-stream. We will also make use of a wake oscillator to describe the lift resulting from vortex shedding. This class of models was originally derived to represent the dynamics of the free wake behind a fixed structure (Birkhoff and Zarantonello, 1957; Bishop and Hassan, 1964), and they have been proved able to capture some characteristic features of the vortex shedding mechanism, such as the formation of cells in shear flow (Noack et al., 1991; Mathelin and de Langre, 2005). Such models have also been proved useful in qualitatively describing the physics of VIVs when coupled with a structural oscillator (Hartlen and Currie, 1970; Skop and Balasubramanian, 1997; Balasubramanian et al., 2000; Mukundan et al., 2009; Srinil and Zanganeh, 2012), and they have been validated against experimental and numerical results (Violette et al., 2007). In this regard, the work of Facchinetti et al. (2004a) demonstrated that features such as the boundaries of the lock-in range, the amplitude of the vibrations or the phase between the structure and the wake are correctly predicted when a coupling term proportional to the structural acceleration is used. The subsequent work of de Langre (2006) and Violette et al. (2010) further demonstrated that many features of the nonlinear limit-cycle dynamics can be interpreted through the linear analysis of the coupled wake–structure oscillators.

In Section 2, the model for the flow-structure interactions and its specific adaptations to the problem in question are detailed. The consequences of the flow-induced bending on the VIVs are then discussed in Section 3 based on the results of numerical simulations.

#### 2. Model

#### 2.1. Theoretical modelling

We consider the model system represented on Fig. 1. A circular cylinder of length *L*, diameter *D* and mass per unit length *m* is clamped perpendicular to a uniform and steady flow of velocity  $Ue_x$  of a fluid of density  $\rho$ . We assume the cylinder is slender ( $D \ll L$ ) and we model it as an inextensible Euler–Bernoulli beam of bending stiffness *EI* (see more details about the structural model in Appendix A.1). We note *s* the curvilinear coordinate along the span from the clamped end to the free



**Fig. 1.** (a) Dimensions of the undeformed cylinder and centreline (broken line). (b) 3D view of the centreline of the vibrating cylinder, static deflection  $r_0(s)$  (broken black line), fully deformed shape r(s, t) (solid grey line). The  $\pi$ -plane in grey indicates the surface upon which the vibration occurs. (c) Projection of the deformation on the plane of static deflection (*xz*-plane). (d) Projection of the deformation on the surface or sy-surface).

end, and  $\mathbf{r}(s, t)$  the location of the centreline. In this paper we focus on the small-amplitude transverse oscillations so we consider that the structure is primarily deflected in the *xz*-plane under the effect of the flow into a leading order configuration  $\mathbf{r}_0(s)$  (broken black line on Fig. 1, specifically shown on Fig. 1(c)). Vortex shedding in the wake of the bent structure is then responsible for an oscillatory lift force in the *y*-direction that induces small-amplitude transverse vibrations (solid grey line on Fig. 1, specifically shown on Fig. 1, specif

We disregard the influence of gravity and buoyancy, and we restrict our study to large Reynolds number flows so that friction forces are neglected. Following Mukundan et al. (2009) and Violette et al. (2010), we model the effect of the fluid as the combination of three load distributions along the span  $p = p_{am} + p_d + p_w$ , where  $p_{am}$  is the reactive or added mass force,  $p_d$  is the resistive drag due to flow separation in the plane of the cross-section, and  $p_w$  is the oscillating lift force due to the vorticity wake. The expressions of these forces involve the projections ( $U_\tau$ ,  $U_N$ ) on the direction tangent to the structure and on the plane normal to it, of the relative velocity between the cylinder and the fluid  $U_{rel} = \dot{Y}e_y - Ue_x = U_\tau \tau + U_NN$ , where the overdot stands for time differentiation.

Firstly, in the case of an elongated body such as those considered in this study, the expression of the added mass force stemming from the pressure field associated with the potential component of the flow has originally been derived by Lighthill (1971) for large-amplitude planar motions. Its extension to 3D deformations has been derived by Candelier et al. (2011) and reads

$$\boldsymbol{p_{am}} = -m_a \left[ \partial_t (U_N \boldsymbol{N}) - \partial_s (U_\tau U_N \boldsymbol{N}) + \frac{1}{2} \partial_s (U_N^2 \boldsymbol{\tau}) \right]$$
(1)

with the added mass  $m_a = 1/4\rho\pi D^2$ . This expression differs from the usual added mass force used in VIV studies because it is designed to account for the effects of the curvature as well as a large axial flow component. The additional terms thus introduced cancel in the specific case of a straight structure so this choice is nonetheless consistent with the usual expression used in previous works on the topic. On the other hand, this expression has been widely used in studies of the flutter instability of slender beams in axial flow (Païdoussis, 1998; de Langre et al., 2007; Eloy et al., 2012; Singh et al., 2012b; Michelin and Doaré, 2013).

Secondly, following Taylor (1952), we express the resistive drag force as

$$\boldsymbol{p}_{\boldsymbol{d}} = -\frac{1}{2}\rho C_{D}D|\boldsymbol{U}_{N}|\boldsymbol{U}_{N}\boldsymbol{N}$$
<sup>(2)</sup>

with  $C_D$  the cross-section drag coefficient. Due to the transverse vibrations, this coefficient may be dynamically enhanced by an amplitude-dependent factor classically of the order of 2 (Vandiver, 1983; Blevins, 1990; Chaplin et al., 2005). The exact value of the drag coefficient should however have only a limited impact on the qualitative features of the VIVs. For the sake of simplicity, we hereafter take  $C_D = 2$  for a vibrating cylinder (Blevins, 1990; Chaplin et al., 2005; Mathelin and de Langre, 2005). The resistive drag term is responsible for the leading order static deflection of the beam in the *xz*-plane (see Gosselin et al., 2010; Leclercq and de Langre, 2016) and acts as hydrodynamic damping for the transverse vibrations.

Finally, following Antoine et al. (2016) the local effect of vortex shedding is modelled as an oscillating lift force orthogonal to the cylinder axis and to the component of the free-stream normal to the still cylinder

$$\boldsymbol{p}_{\boldsymbol{w}} = \frac{1}{4} \rho C_L^0 D U^2 \cos^2 \theta_0 \, \boldsymbol{q} \, \boldsymbol{e}_{\boldsymbol{y}} \tag{3}$$

where the fixed cylinder lift coefficient  $C_L^0$  is multiplied by a local magnification factor q(s, t) that satisfies a nonlinear Van der Pol equation forced by the transverse acceleration of the structure

$$\ddot{q} + \varepsilon \left(2\pi \frac{S_t U}{D} \cos \theta_0\right) \left(q^2 - 1\right) \dot{q} + \left(2\pi \frac{S_t U}{D} \cos \theta_0\right)^2 q = A \frac{\ddot{Y}}{D}.$$
(4)

We assume in this study the classical values  $S_t = 0.2$ ,  $C_L^0 = 0.3$  in the sub-critical range  $300 < R_e < 1.5 \times 10^5$  (Blevins, 1990; Facchinetti et al., 2004a). Following Facchinetti et al. (2004a), we further assume A = 12,  $\varepsilon = 0.3$ . Even though these parameters were derived from experiments on rigid cylinders, this approach was validated in the case of flexible structures in Facchinetti et al. (2004b) and Violette et al. (2007). It was then successfully used in subsequent theoretical studies (Mathelin and de Langre, 2005; Violette et al., 2010; Meng and Chen, 2012; Dai et al., 2013, 2014). No direct spanwise coupling between the wake oscillators is considered in Eq. (4) as the work of Mathelin and de Langre (2005) has demonstrated that the coupling with the structural oscillator is the main source of synchronization in the wake. Similar observations have been reported by de Vecchi et al. (2008) from direct numerical simulations that showed parallel vortex shedding in the wake of a curved cylinder in forced oscillations, in contrast with the spanwise phase shift reported by Miliou et al. (2007) in the wake behind a fixed cylinder. Moreover, this local modelling is based on the so-called independence principle that states that the physics of vortex shedding is primarily governed by the component of the free-stream normal to the structure only (see Lucor and Karniadakis, 2003; Franzini et al., 2009; Jain and Modarres-Sadeghi, 2013). The work of Bourguet et al. (2015) has demonstrated the validity of that principle when applied locally at each location along the span in the case of the VIVs of flexible cylinders inclined at 60° of incidence. In this case, due to the in-line deformation of the structure, the local incidence of the cylinder could reach values as large as 75°. The local lift force (3) is thus taken quadratic in the locally normal projection of the free-stream velocity U cos  $\theta_0$  and the local natural shedding frequency that appears in (4) satisfies the Strouhal law expressed in terms of that normal component as well  $f_w = S_t U \cos \theta_0 / D$  (Williamson, 1996; Facchinetti et al., 2004b).

Note that vortex shedding in the wake is also responsible for an oscillating drag term that creates slight temporal variations of the in-line deflection. Besides, recent studies have shown that the coupling between the in-line and transverse vibrations may considerably affect the transverse dynamics (Jauvtis and Williamson, 2003, 2004; Marcollo and Hinwood, 2006; Srinil and Zanganeh, 2012; Srinil et al., 2013). However, for the sake of simplicity and given the leading-order nature of the resistive drag term considered here, we choose to disregard the influence of the in-line fluctuations in this study.

#### 2.2. Governing equations

We non-dimensionalize all variables but the transverse displacement using the length *L* of the cylinder as characteristic length, and the inverse of the natural shedding frequency on the undeformed straight cylinder  $f_w^0 = S_t U/D$  as characteristic time scale. Because the amplitude of the VIVs classically scales with the diameter of the structure, we non-dimensionalize the transverse displacement *Y* using *D* instead of *L*. We also define the following non-dimensional parameters: the Cauchy number  $C_Y$ , the reduced velocity *u*, the aspect ratio  $\Lambda$ , the mass ratio  $\beta$ 

$$C_{\rm Y} = \frac{\rho C_{\rm D} D U^2 L^3}{2EI}, \qquad u = \frac{S_t U}{D} L^2 \sqrt{\frac{m + m_a}{EI}}, \qquad \Lambda = \frac{L}{D}, \qquad \beta = \frac{m_a}{m + m_a}$$
(5)

and rescaled drag and lift coefficients  $c_d = 2C_D/\pi$  and  $c_l = C_L^0/\pi$ . The Cauchy number measures the level of static in-plane deflection. The definition of the Cauchy number used here is classical in studies of the static deflection of slender structures (Gosselin et al., 2010; Leclercq and de Langre, 2016; Hassani et al., 2016). More details about the significance of this number will be provided in Section 2.3. Note that the reduced velocity used here is consistent with the classical definition used in VIV studies of systems with only one degree of freedom (see for instance Khalak and Williamson, 1999; Païdoussis et al., 2010), in the sense that it compares the characteristic scale of the natural period of the structure  $T_s = L^2 \sqrt{(m + m_a)/EI}$  to the reference vortex shedding period  $1/f_w^0 = D/S_t U$ . Our definition of the mass ratio on the other hand differs from the more classical ratio  $m^* = m/(\rho \pi D^2/4)$  (Païdoussis et al., 2010) and they are related by  $m^* = 1/\beta - 1$ . The proper linearization of the model presented in Section 2.1 is detailed in Appendix A, and its appropriateness for the modelling of VIVs is discussed in Appendix B. Note that, for the sake of simplicity we do not introduce different notations for the non-dimensional variables

and in the rest of this paper, the notations Y,  $\kappa$ , s and t always refer to the non-dimensional quantities. In terms of the non-dimensional variables, the governing equation for the leading order static deflection in the xz-plane then reads

$$\kappa_0'' + \frac{1}{2}\kappa_0^3 + \frac{C_Y}{c_d\Lambda} \left(S_0^2 - \frac{1}{2}C_0^2\right)\kappa_0 - C_Y C_0^2 = 0$$
(6)

where the prime notation stands for curvilinear differentiation, the notations  $S_0 = \sin \theta_0$  and  $C_0 = \cos \theta_0$  have been used for brevity, and with the curvature  $\kappa_0 = \theta'_0$ . The corresponding boundary conditions are  $\theta_0 = 0$  at the clamped edge s = 0and  $\kappa_0 = \kappa'_0 = 0$  at the free tip s = 1. At the linear order, the equation for the non-dimensional vibration Y(s, t) is

$$u^{2}\ddot{Y} + \frac{\beta u^{2}}{S_{t}\Lambda} \left[ 2S_{0}\dot{Y}' + C_{0} \left(\kappa_{0} + c_{d}\Lambda\right)\dot{Y} \right] + \left( \left[ \frac{\beta u^{2}}{S_{t}^{2}\Lambda^{2}} \left( S_{0}^{2} + \frac{1}{2}C_{0}^{2} \right) + \frac{3}{2}\kappa_{0}^{2} \right] Y' \right)' + c_{d}\frac{\beta u^{2}}{S_{t}^{2}\Lambda}C_{0}S_{0}Y' + Y^{(4)} = c_{l}\frac{\beta u^{2}}{S_{t}^{2}}C_{0}^{2}q$$

$$\tag{7}$$

with Y = Y' = 0 at the clamped edge s = 0 and Y'' = Y''' = 0 at the free tip s = 1. The lift magnification factor q(s, t) then satisfies the Van der Pol equation

$$\ddot{q} + \varepsilon (2\pi C_0) \left( q^2 - 1 \right) \dot{q} + (2\pi C_0)^2 q = A \ddot{Y}.$$
(8)

#### 2.3. On the governing parameters

Within the four non-dimensional parameters defined above ( $C_Y$ , u,  $\Lambda$ ,  $\beta$ ), only three are necessary to fully characterize the problem. In fact, for a given structure with a fixed mass and aspect ratios, the Cauchy number and the reduced velocity are redundant parameters that are related by  $C_Y = (c_d \beta / S_t^2 \Lambda) u^2$ . Both quantities scale the influence of the flow to the rigidity of the structure, but from different perspectives. The Cauchy number expresses the ratio between the leading order drag force and the restoring structural stiffness (Tickner and Sacks, 1969; Chakrabarti, 2002; de Langre, 2008). Consequently, it controls the level of static deflection in the *xz*-plane. For small  $C_Y < 1$ , deflection is negligible and the structure stands upright in the flow, while for  $C_Y \gg 1$  the cylinder is highly deformed in the direction of the free-stream (Gosselin et al., 2010). On the other hand, the reduced velocity compares the natural shedding frequency  $f_w^0$  (on the straight structure) to the scale of the natural period of the structure  $T_s$ , so it controls the dynamic behaviour of the coupled wake–structure system.

The mass ratio  $\beta$  represents the amount of fluid inertia within the total inertia of the system. We do not expect the precise value of  $\beta$  should significantly influence the qualitative features of the VIVs presented in this article. We also expect the influence of the gravity and buoyancy forces on the static deflection due to the flow to remain small (Luhar and Nepf, 2011). Thus, for the sake of simplicity, we restrict this study to the case of a neutrally buoyant cylinder  $\beta = 0.5$ . Finally, the aspect ratio  $\Lambda = L/D$  scales the relative contributions of the resistive and reactive terms. In the case of slender structures  $\Lambda \gg 1$  as those considered in this paper, the contribution of the reactive force  $O(C_Y/\Lambda)$  to the static equilibrium shape in Eq. (6) is in fact negligible. On the other hand, as will be shown in Section 3.3, the reactive force may be responsible for triggering the large-amplitude flutter instability, and its influence on the vortex-induced dynamics may still be significant near the critical threshold when the deflection is large. Thus we may not neglect a priori the terms originating from the reactive force in Eq. (7).

Our main interest in this study is the effect of the flow-induced bending on the properties of the vortex-induced vibrations. In other words, we wish to compare the dynamics of structures with varying levels of deflection in the direction of the freestream (varying Cauchy numbers  $C_Y$ ), at a given reduced velocity u. However, as explained above, for a given fluid–structure system of fixed mass and aspect ratios ( $\beta$ ,  $\Lambda$ ), the Cauchy number and the reduced velocity vary together as  $C_Y \propto \beta u^2/\Lambda$ . Therefore, increasing the Cauchy number *with a fixed reduced velocity* requires to change the structure to lower the aspect ratio  $\Lambda$ . This may seem counter-intuitive, as a lower aspect ratio is classically associated with lower deflections. Indeed, the Cauchy number is an increasing function of the aspect ratio  $C_Y \propto \Lambda^3$  when the dimensional flow velocity, fluid density, and Young's modulus of the solid material are kept constant. In our case however, we allow these dimensional quantities to vary in order to keep the reduced velocity u constant instead, and consequently the Cauchy number becomes a decreasing function of the aspect ratio. In the limit of an infinitely slender structure  $\Lambda \rightarrow +\infty$ , the Cauchy number remains much smaller than 1 even for arbitrarily large reduced velocities. This asymptotic case is therefore equivalent to considering a structure that remains straight in the free-stream. For this benchmark case, the dynamic system (7)–(8) reduces to

$$u^{2}\ddot{Y} + c_{d}\frac{\beta u^{2}}{S_{t}}\dot{Y} + Y^{(4)} = c_{l}\frac{\beta u^{2}}{S_{t}^{2}}q$$
(9)

$$\ddot{q} + \varepsilon \left(2\pi\right) \left(q^2 - 1\right) \dot{q} + \left(2\pi\right)^2 q = A \ddot{Y}$$
<sup>(10)</sup>

which is the standard system of equations for the modelling of VIVs by means of a wake oscillator, in the case of a straight slender cylinder with flexural stiffness and hydrodynamic damping (Mukundan et al., 2009).



**Fig. 2.** (a) Deflection in the *xz*-plane and (b) normal velocity profile normalized by the horizontal velocity, in the straight case (grey – ), and in deflected cases  $C_Y = 10^0 (\cdots)$ ,  $C_Y = 10^1 (---)$ , and  $C_Y = 10^2$  (black – ) for  $\Lambda = 10^3$ .

#### 3. Numerical results

In order to analyse the vibration behaviour, we solve the problem defined by Eqs. (6), (7), (8) numerically. The beam is discretized using N = 100 Gauss–Lobatto points  $s_k = \frac{1}{2}(1 - \cos((k - 1)/(N - 1)\pi))$ , and the curvilinear derivatives are computed by Chebyshev collocation. For a given aspect ratio  $\Lambda$ , we first compute the static solution of Eq. (6) iteratively by increasing the Cauchy number from the upright case  $C_Y = 0$ . After each increment, we solve Eq. (6) with a pseudo-Newton solver (method of Broyden, 1965) using the solution at the previous step as initial guess. Then, for each value of  $C_Y$  (corresponding to a given u), we solve the nonlinear system (7)–(8) using a time-stepping method, with time step  $dt = 10^{-2}$ . The time derivatives are computed using implicit second order accurate finite differences. At each time step, the nonlinear boundary value problem involving the unknown spanwise distributions (Y, q) is solved thanks to the pseudo-Newton solver. The simulations are run for 200 periods and the last 100 periods are considered for the analysis in order to cut the transient.

#### 3.1. Wake excitation bandwidth and inhibition of single mode lock-in

First, we discuss the influence of the deflection on the modal content of the dynamics. To do so, we compare the vibrations of a structure that bends (finite aspect ratio  $\Lambda = 10^3$ ) to that of a structure that remains straight ( $\Lambda \to +\infty$ ), on the same range of reduced velocities. For the structure with finite aspect ratio  $\Lambda = 10^3$ , the variations of *u* within the range considered will be associated with variations in the Cauchy number  $C_Y \propto u^2/\Lambda$  leading to varying levels of deflections. On the other hand, the infinitely slender cylinder will remain straight because the Cauchy number will remain asymptotically small in the same finite range of *u*. We recall here that the cylinder with the finite aspect ratio is the one that deflects while the infinitely slender one remains unbent because we compare the dynamics of the two structures *on the same range of reduced velocities* (see Section 2.3 for more details).

As explained in Section 2.1, the independence principle states that the natural frequency of vortex shedding  $f_w(s)$  decreases along the span when the structure is deformed because of the projection of the free-stream on the normal direction  $f_w(s)/f_w^0 = U_{n_0}/U = \cos \theta_0(s)$ . The deforming structure is therefore subjected to a forcing by the wake on a continuum of frequencies that broadens when the deflection is enhanced with increasing Cauchy numbers, as shown on Fig. 2.

The enrichment of the excitation spectrum significantly alters the modal content of the structural response. This is illustrated on Fig. 3 for the vibration spectrum and Fig. 4 for the spanwise localization of the vibration. In the straight case, the structure is subject to a forcing by the wake at a single frequency along the whole span. The frequency of the vibration remains close to the Strouhal law ( $f \simeq 1$  on Fig. 3(a)), but slightly deviates to follow an evolution closer to that of the nearest structural mode. As u varies, lock-in with the successive structural modes occurs in turn and frequency discontinuities mark the transitions between consecutive lock-ins. This phenomenon is a well-known feature of the VIVs of flexible structures perpendicular to the flow (King, 1995; Chaplin et al., 2005; Violette et al., 2010). As expected, this behaviour remains unchanged for as long as the deflection is negligible ( $C_Y < 1$  on Fig. 3(b)) in the case  $\Lambda = 10^3$ . However, at the early stages of reconfiguration ( $C_Y \leq 10^1$  on Fig. 3(b)), the ranges of reduced velocities within which lock-in with a given mode occurs are slightly widened and increasingly shifted towards larger reduced velocities (see Table 1). For instance, lock-in with mode 4 persists on a slightly larger range  $u \in [16.3, 29.9]$  when the structure is bent compared to  $u \in [15.2, 27.0]$  when it is not. The modal shapes in both configurations remain on the other hand almost identical, as illustrated for u = 19.8 in Fig. 4(c), which proves that the curvature of the structure in the *xz*-plane only has a very limited impact on the transverse deformation.

Following the method of Violette et al. (2010), we may interpret these results by comparing the spectrum of the nonlinear dynamics to the frequencies of the eigenmodes found by a linear stability analysis (see Fig. 5). The details of the linear

Table 1

educed velocity ranges	for lock-in in the	straight and bent c	cases (nonlinear and	l linear ranges).
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	Straight case (nonlinear)	Bent case $\Lambda = 10^3$ (nonlinear)	Bent case $\Lambda = 10^3$ (linear)
mode 1	0-2.3	0-2.3	0-2.16
mode 2	2.4-6.8	2.4-6.9	2.16-6.80
mode 3	6.9-15.1	7–16.2	6.80-15.86
mode 4	15.2–27.0	16.3–29.9	15.86-28.70

stability analysis can be found in Appendix C. The eigenmodes thus found can be classified in 3 distinct types. Firstly, a series of N unstable modes (where N is the number of discretization points  $s_k$ ) with eigenvalues matching almost exactly the eigenvalues of the linearized wake oscillator without structural coupling  $\omega_k \simeq \cos\theta_0(s_k)(2\pi\sqrt{1-(\varepsilon/2)^2-i\pi\varepsilon})$  are observed. They have no significant structural component ( $\phi_k(s) \simeq 0$ ), and their wake components are localized respectively at each discretization point  $(|\psi_k(s)| \simeq \delta(s - s_k))$  with  $\delta$  the Dirac function). These modes thus correspond to the naturally unstable free wake modes. For the sake of clarity, the continuous spectrum of the free wake oscillator itself is represented in grey on Fig. 5(b) instead of the N individual eigenmodes spanning the area. Secondly, the modes in blue correspond closely to the eigenmodes of the structural equation (7) without coupling with the wake. These modes may thus be designated as the free structural modes. Finally, the four modes in black are coupled modes that each arise from lock-in with one of the structural modes. They are all unconditionally unstable and will be referred to as the lock-in modes. Following Violette et al. (2010), we expect lock-in with a given mode to persist in the nonlinear limit-cycle if the corresponding linear lock-in mode is the most unstable. The comparison of the linear and nonlinear lock-in ranges in Table 1 confirms these findings and we may thus lean on the linear analysis to interpret the dynamics observed in the limit-cycle. In particular, as deflection increases, we notice on Fig. 5 that the broadening of the free wake spectrum allows the structural frequencies to remain within the excitation bandwidth on a larger range of reduced velocities. Lock-in with these modes may consequently occur on these larger ranges as well. Hence, it appears that the primary consequence of the deflection is the broadening of the free wake spectrum due to the increased shear in the normal component of the free-stream. At leading order, a bending structure in a uniform flow is therefore equivalent to a straight structure in a sheared flow whose profile varies, depending on the Cauchy number, according to that of the normal flow shown on Fig. 2(b).

When bending is more pronounced ( $C_Y > 10^1$ ), not only one but several frequencies are involved in the spectrum of the nonlinear limit-cycle Fig. 3(b), and the localization of the transverse vibration significantly deviates from the single-mode shape on Fig. 4. Indeed, when the wake spectrum becomes large enough, several structural modes are excited simultaneously at different locations along the span (several structural modes appear within the wake excitation bandwidth on Fig. 5). Consequently, single mode lock-in is replaced by a multi-frequency response of the whole structure associated with the spatial fragmentation of the wake into multiple cells of locally uniform frequency. As illustrated on Fig. 6, the frequency within each wake cell matches that of the structural mode that is closest (in the frequency space) from the Strouhal law, so that it might be said that single mode lock-in is actually replaced by multiple occurrences of lock-in along the span. It is however noteworthy that the local wake dynamics is essentially monochromatic, while the structural dynamics involves comparable contributions from all the excited modes at any location along the span. Besides, the linear analysis shows that, past u > 28.7, the most unstable mode is not one of the coupled lock-in modes anymore, but is found instead within one of the free wake modes. No fifth coupled mode that would appear because of lock-in, but leads to the simultaneous excitation of multiple structural modes that all participate in the dynamics.

We may finally conclude that bending primarily affects the dynamics through its influence on the component of the free-stream normal to the cylinder. More specifically, the growing shear in the normal flow is responsible for broadening the wake spectrum. When the deflection remains moderate, the wake excitation bandwidth remains narrow enough so that single-mode lock-in still prevails, but on a slightly larger range of reduced frequencies. On the other hand, when the deflection is large, several structural modes may be simultaneously excited, leading to the inhibition of single mode lock-in replaced by a multi-frequency response to the broadband excitation. These conclusions are consistent with the observations of Bourguet et al. (2012, 2015) for a pinned-pinned cylinder experiencing small in-line deformations, as well as those of Vandiver (1993), Ge et al. (2011) and Srinil (2011) for straight pinned-pinned cylinders in shear flow.

#### 3.2. Localization of the excitation and VIV mitigation

As emphasized in the introduction, the experimental work of Assi et al. (2014) and Seyed-Aghazadeh et al. (2015) has shown that the amplitude of the vibrations of rigid cylinders is much reduced under the effect of the curvature. The same observation was made by Bourguet et al. (2015) regarding the VIVs of pinned–pinned flexible cylinders. Similarly, the works of Trim et al. (2005), Ge et al. (2011) and Srinil (2011) have also shown that sheared incoming flows entailed lower structural responses than uniform flows. Our numerical simulations indicate that this observation holds true as well for flexible cantilever cylinders curved by the flow. Indeed, the amplitude of the VIVs of the deflecting cylinder ( $\Lambda = 10^3$ ) reduces progressively on Fig. 7(a) compared to the straight case above the bending threshold  $C_Y > 1$ . For large deflections  $C_Y > 10$ , the amplitude settles around approximately 1/3 of the amplitude of the straight case.



**Fig. 3.** Comparison between the power spectral densities of the motion at the tip Y(s = 1, t) in (a) the straight case and (b) the deflected case  $\Lambda = 10^3$ , for varying reduced velocities *u*. The natural wake excitation bandwidth (WEB) is superimposed. The particular values of the reduced velocity *u* corresponding to Fig. 2 are indicated as well.



**Fig. 4.** Comparison between the spanwise localizations of the transverse deformation in (a) the straight case and (b) the deflected case  $\Lambda = 10^3$ , for varying reduced velocities *u*. The colorplots show the temporal RMS of *Y*(*s*, *t*), normalized to 1 along the span. The particular values of the reduced velocity *u* corresponding to Fig. 2 are indicated as well. (c) Comparison of the envelopes of the transverse deformation in the straight case (---) and deflected case  $\Lambda = 10^3$  (-), for particular values of *u* corresponding to successive structural modes in the straight case.

We may explain this reduction of amplitude by considering the energy transferred to the structure from the wake oscillator. In non-dimensional form, the work of the oscillating lift force at a given location along the span reads  $e = \dot{Y} \cdot q \cos^2 \theta_0$ , so that the total energy *E* transferred to the structure over one cycle of oscillations is the temporal mean of *e*, integrated over the whole span. The evolution of *E* with the reduced velocity on Fig. 7(b) proves that deflection drastically mitigates the transfer. A more detailed look at the spanwise distribution of the mean energy transfer on Fig. 8 indicates that the excitation by the wake, which is equally distributed in the straight case, concentrates around  $s \sim 0.2$  as the deflection



**Fig. 5.** Evolution of the linear frequencies of the coupled wake–structure system with the reduced velocity u, for (a) the straight case and (b) the deflected case  $\Lambda = 10^3$ . The unconditionally unstable free wake spectrum is represented by the grey area. The structural modes in blue are always stable. The coupled modes, displayed in black, are always unstable. The most unstable mode for a given range of u is emphasized in bold.



**Fig. 6.** Spanwise distribution of power spectral density of (a) the structural motion Y and (b) the wake oscillator q, for  $\Lambda = 10^3$  and u = 49.0. The natural shedding frequency  $f_w^0(s)$  given by the Strouhal law is superimposed in (b) (–).

increases. This is actually quite intuitive, because the lift force varies with the square of the normal component of the freestream  $\propto \cos^2\theta_0$ . When deflection is important, only the small region close to the clamping point remains close enough to the vertical so as to significantly contribute to the excitation. As the Cauchy number increases and the cylinder bends more and more, the size of that region reduces progressively. The amount of energy transferred overall is consequently reduced, and the amplitude of the vibrations accordingly mitigated.

Note that the discontinuities observed in the straight and low-deflection cases on Fig. 7 correspond to mode switches. The disappearance of these jumps above u > 30 in the deflected case is consistent with the continuous evolution of the spectrum and vibration shape reported in Figs. 3(b) and 4(b).



**Fig. 7.** Comparison of (a) the RMS amplitude of vibration  $Y_{rms}$  and (b) the total energy transfer *E* from the wake to the structure, between the straight case (blue  $\circ$ ) and the deflected case  $\Lambda = 10^3$  (orange  $\nabla$ ) for varying reduced velocities *u*. The particular values of the reduced velocity *u* corresponding to Fig. 2 are indicated as well.



**Fig. 8.** Comparison of the spanwise distributions of energy transfer from the wake to the structure between (a) the straight case and (b) the deflected case  $\Lambda = 10^3$ , for varying reduced velocities *u*. The colorplots show the mean work of the oscillating lift force  $\langle e \rangle$  normalized by the total energy transfer *E*. The particular values of the reduced velocity *u* corresponding to Fig. 2 are indicated as well.

#### 3.3. Influence of the reactive force

Finally, we discuss the influence of the large axial component of the free-stream when the structure is highly reconfigured. Indeed, as the flow velocity increases, the inclination of the structure switches from perpendicular to mostly parallel to the flow. As explained in Section 2.3, the level of deflection is controlled by the Cauchy number  $C_Y \propto u^2/\Lambda$ . We focus here on a case where the deflection increases more rapidly with the reduced velocity than before, that is to say a structure with a smaller aspect ratio  $\Lambda = 10$ .

At first, the amplitude of vibrations on Fig. 9(a) follows the same trend as in the previous case: below  $C_Y < 1$ , the effect of bending is unnoticeable, while it results in a constant amplitude much reduced compared to the straight case when bending is significant. The amplitude is even reduced as low as 1/10th of the straight case. But conversely to the previous case, the vibrations start growing again slowly past  $u \sim 20$  and they even exceed the amplitude of the straight case for  $u \gtrsim 33.5$ . Above some critical threshold  $u \sim 35$ , the amplitude finally grows continuously during the whole time of the simulation. The VIVs are by nature a self-initiated and self-limited phenomenon. In the model, the onset of the VIVs is ensured by the



**Fig. 9.** (a) Comparison of the RMS amplitude of vibration between the straight case (blue  $\circ$ ) and the deflected case  $\Lambda = 10$  (yellow  $\Box$ ). (b) and (c) Linear frequencies  $f_{lin}$  and growth rates  $\sigma$  of the coupled wake–structure system, for  $\Lambda = 10$ . The unconditionally unstable free wake spectrum is represented by the grey area. The structural modes are in blue when stable, and in bold orange when unstable. The orange cross marks the stability threshold for structural mode 3. The coupled mode, displayed in black, is always unstable. It is emphasized when it is the most unstable mode.

negative damping of the Van der Pol wake oscillator when the amplitude of *q* is small, while the limitation of the amplitude is ensured by the nonlinear saturation of that same term. This unbounded growth is therefore not related to the VIVs, but is instead the consequence of the onset of a flutter instability caused by the destabilizing influence of the hydrodynamic reactive force on the large portion of the cylinder that is aligned with the free-stream (Eloy et al., 2007; Singh et al., 2012a).

This is confirmed by the results of the linear stability analysis presented on Fig. 9(b) and (c). Lock-in with structural mode 1 occurs for the smallest reduced velocities (the coupled linear mode is the most unstable as long as u < 2.30), when flow-induced bending is still small. When deflection becomes significant ( $C_Y \gg 1$ ), several structural modes are simultaneously excited by the wake on Fig. 9(b). But in this case, one of the structural modes (mode 3) is progressively destabilized as u increases. Above  $u \gtrsim 20$ , the growth rate of this mode starts increasing on Fig. 9(c) until it finally becomes unstable at the critical threshold  $u_c = 34.9$  (marked by the orange cross on Fig. 9(b) and (c)). If the effect of the curvature on the structural modes has been proved negligible in Section 3.1, these observations prove on the other hand that the influence of the reactive force may be very significant in the vicinity of the critical threshold. The progressive increase of the vibration amplitude from  $u \sim 20$  until the actual onset of the instability  $u_c = 34.9$  is indeed concomitant with the destabilization of structural mode 3. Hence, the larger vibration amplitudes observed in this range of reduced velocities are most likely attributable to the growing influence of the gradually destabilizing structural mode 3.

The flow-induced bending thus has competing consequences on the amplitude of the vibrations. On the one hand, we have shown in Section 3.2 that the shrinkage of the wake excitation zone considerably mitigates the VIVs. We demonstrate

here that on the other hand, the reorientation of the structure in the direction of the free-stream may amplify the vibrations because of the destabilizing effect of the reactive force on the structural modes. This last effect becomes significant in the vicinity of the structural stability threshold. As explained in Appendix D, the structural stability threshold in terms of either the reduced velocity u or the Cauchy number  $C_Y$  is close to proportional to the aspect ratio  $\Lambda$ . More slender structures may consequently reach much higher modes and larger deflections before feeling the destabilizing influence of the reactive force.

#### 4. Discussion and conclusion

In this paper, we have provided a qualitative analysis of the consequences of flow-induced bending on the vortex-induced dynamics of slender flexible cylinders based on reduced order models. Overall, the effects of the deflection may impact the features of the vibrations on two levels.

Firstly, the deformation of the cylinder changes the spanwise profile of the component of the free-stream normal to the structure. We have shown that the increasing non-uniformity of the normal flow leads to the spreading of the wake excitation spectrum and the localization of the energy transfer from the wake to the structure, owing to the independence principle. These two effects result in a strong mitigation of the amplitude of the vibrations coupled with the inhibition of single mode lock-in replaced by the simultaneous excitation of multiple structural modes. These mechanisms were indeed evidenced in several experimental and numerical studies about the VIVs of straight cylinders in shear incoming flow, such as Vandiver (1993), Trim et al. (2005), Srinil (2011), Ge et al. (2011) and Bourguet et al. (2013) for instance. As any deformation of a straight cylinder in the direction of the free-stream would induce shear in the normal flow, we expect that these conclusions are independent of the initial structural configuration, boundary conditions, or the features of the in-plane deformation. For instance, similar observations have been reported by Bourguet et al. (2015) for a pinned-pinned cylinder initially inclined and slightly deformed by the flow. Similarly, the smaller amplitudes of vibrations reported in Assi et al. (2014) and Seyed-Aghazadeh et al. (2015) for rigid curved cylinders compared to straight ones is likely also the consequence of the difference in the normal flow profiles between the two configurations. The loss of harvesting efficiency reported in Antoine et al. (2016) when increasing the sag (and consequently the deformation) in the in-flow catenary configuration also results from the induced shear in the normal flow.

Secondly, the deformation of the structure is responsible for a modification of the structural modes resulting from both the flow-induced tension and the axial component of the flow through the reactive forcing term.

The effect of the tension is negligible in our case, but it should be noted that it may be of significant importance in other situations. Indeed, the structural frequencies depend on the total stiffness of the system, which can be decomposed in the natural bending stiffness *EI* specific to the structure, and an additional tension-induced stiffness. In our case, the structural tension  $T = -1/2EI\kappa_0^2$  itself is the physical consequence of the structural stiffness *EI*, and so it is understandable that the influence of the tension-induced stiffness on the natural frequencies of the structure be small compared to that of the natural stiffness that originated it. However, in the rather common case of tensioned cables, the structural tension is either externally controlled or flow-induced but it is in any case independent of the usually small bending stiffness. Its influence on the structural frequencies and thus on the vortex-induced dynamics may then be totally dominant (see for instance the study of Antoine et al., 2016), but this effect is out of the scope of the present work as the tension in this case is not related to the flow-induced deformation of the structure.

The effect of the axial flow component may on the other hand be dramatic in the configuration chosen in this article. When the deflection is very large, the axial flow may become the dominant component, and the added damping and stiffness stemming from the reactive forcing on the inclined portion of the cylinder may significantly alter the structural modes. Above some critical velocity threshold, the cylinder might undergo some large-amplitude oscillations originating from a flutter instability. It should be noted that only the transverse stability along the y-direction has been considered in this article, but flutter may also be initiated in the xz-plane (see Leclercq et al., 2018). However, we do not expect the in-plane destabilization to have consequences on the vortex-induced dynamics below the stability threshold. Indeed, the geometrical linearization performed in Appendix A.2 results in the decoupling of the small-amplitude dynamics in the two directions. There should consequently be no interactions between the in-plane structural modes and the transverse vibrations as long as the amplitude of vibration remains small  $Y/L \ll 1$ . On the other hand, we have shown that the progressive destabilization of the transverse structural modes may enhance the amplitude of the VIVs even in the stable domain. These conclusions may not easily be generalized to other structural configurations as the stability of the structural modes is highly dependent on the boundary conditions of the structure. The choice of the cantilever configuration in our work merely demonstrates the potentially large consequences the axial flow may bear through the reactive force, and neglecting the terms originating from it must only be done with great care. Nonetheless, we also found that the magnification of the vibrations due to the reactive force becomes significant for higher modes of vibrations and larger levels of deflection as the structure is made more slender. In this regard, it may thus be considered as a secondary effect that sets a limit to the strong abatement of the vortex-induced vibrations more commonly observed.

We may thus conclude that the primary consequence of flow-induced deformations on the vortex-induced dynamics of flexible cylinders is the hindrance of single mode lock-in, replaced by a multi-frequency response of the structure, and the strong mitigation of the vibrations that rest on the modification of the spanwise profile of the normal component of the free-stream. Neglecting the in-line deformation when assessing the features of the VIVs should therefore generally result in an overestimation of the severity of the vibrations.

Table 2

Structural stability thresholds in terms of the flutter-specific reduced velocity v, VIV-specific reduced velocity u, and Cauchy number Cy.

	Critical velocity $v_c$	Critical velocity $u_c$	Critical Cauchy number $C_{Y,c}$
$\Lambda = 10$	12.7	$3.58 \times 10^{1}$	$2.04 \times 10^{3}$
$\Lambda = 10^{2}$	16.1	$4.55 \times 10^{2}$	$3.30  imes 10^4$
$\Lambda = 10^{3}$	17.1	$4.85 \times 10^{3}$	$3.74  imes 10^5$

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#### Appendix A. Derivation of the governing equations

#### A.1. Details about the structural model

Following Audoly and Pomeau (2010), we define a local direct orthonormal frame  $e_{i=1,2,3} = (n, w, \tau)$ , materially attached to the structure and such that  $(n, w, \tau)|_{s=0} = (e_x, e_y, e_z)$ . We also define the Darboux vector  $\Omega = \kappa_n n + \kappa_w w + \gamma \tau$  where  $\kappa_n$  and  $\kappa_w$  are the material curvatures and  $\gamma$  is the twist of the cylinder. By definition, the Darboux vector is such that  $e'_i = \Omega \times e_i$ . For an Euler–Bernoulli beam of bending stiffness *EI* in both (n, w)-directions, torsional stiffness *GJ*, and subjected to an external force q and no external torque, the Kirchhoff equations governing the dynamics of the structure read

$$m\ddot{\mathbf{r}} = \mathbf{F}' + \mathbf{q}, \qquad \mathbf{0} = \mathbf{M}' + \tau \times \mathbf{F} \tag{11}$$

with the internal force vector  $\mathbf{F} = T\mathbf{\tau} + Q\mathbf{n} + P\mathbf{w}$  and the constitutive law for the internal bending moment  $\mathbf{M} = GJ\gamma\mathbf{\tau} + EI\kappa_n\mathbf{n} + EI\kappa_w\mathbf{w}$ . The inextensibility condition reads  $\mathbf{r} = \mathbf{\tau}$ , and for a cantilever beam, the boundary conditions read  $\mathbf{r} = 0$  and  $\mathbf{r}' = \mathbf{e}_z$  at the clamped edge s = 0, and  $\mathbf{F} = \mathbf{M} = 0$  at the free end s = L. Using the second Kirchhoff law (11) and the boundary conditions for the twist, we find that

$$\gamma = 0, \qquad Q = -EI\kappa'_{u}, \qquad P = EI\kappa'_{n} \tag{12}$$

so that finally the curvilinear derivatives of the material frame simplify in

$$\boldsymbol{\tau}' = \kappa_w \boldsymbol{n} - \kappa_n \boldsymbol{w}, \qquad \boldsymbol{n}' = -\kappa_w \boldsymbol{\tau}, \qquad \boldsymbol{w}' = \kappa_n \boldsymbol{\tau} \tag{13}$$

and the curvilinear derivative of the internal force vector that appears on the right hand side of the first Kirchhoff equation (11) reads

$$\mathbf{F}' = \left(T + \frac{1}{2}EI\left(\kappa_n^2 + \kappa_w^2\right)\right)' \mathbf{\tau} + \left(\kappa_w T - EI\kappa_w''\right)\mathbf{n} - \left(\kappa_n T - EI\kappa_n''\right)\mathbf{w}.$$
(14)

The boundary conditions read  $\mathbf{r} = \mathbf{\tau} = \mathbf{0}$  at s = 0 and  $T = \kappa_n = \kappa'_n = \kappa_w = \kappa'_w = 0$  at s = 1.

#### A.2. Linearization of the structural model

As explained in Section 2, we expand  $\mathbf{r}(s, t) = \mathbf{r}_0(s) + Y(s, t)\mathbf{e}_y$  where the transverse displacement Y is taken as a first order perturbation to the static shape  $\mathbf{r}_0$ . At the leading order the deformation is contained exclusively in the *xz*-plane so  $\kappa_n$  is a first order perturbation as well and we also expand  $\kappa_w = \kappa_0 + \delta \kappa$  and the frame vectors

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \delta \boldsymbol{\tau}, \qquad \boldsymbol{n} = \boldsymbol{n}_0 + \delta \boldsymbol{n}, \qquad \boldsymbol{w} = \boldsymbol{e}_{\boldsymbol{v}} + \delta \boldsymbol{w}. \tag{15}$$

Using (13), the inextensibility condition and all the expansions above mentioned, we obtain after some calculations the expansions of the material frame

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{\mathbf{0}} + \boldsymbol{Y}' \boldsymbol{e}_{\mathbf{y}}, \qquad \boldsymbol{n} = \boldsymbol{n}_{\mathbf{0}} + \alpha \boldsymbol{e}_{\mathbf{y}}, \qquad \boldsymbol{w} = \boldsymbol{e}_{\mathbf{y}} - \boldsymbol{Y}' \boldsymbol{\tau}_{\mathbf{0}} - \alpha \boldsymbol{n}_{\mathbf{0}}$$
(16)

where the small angle  $\alpha$  and the small curvature  $\kappa_n$  satisfy

$$\alpha' = -\kappa_0 Y', \qquad \kappa_n = \kappa_0 \alpha - Y'' \tag{17}$$

and  $\delta \kappa_w = 0$ . Besides, expanding all the terms in (14), we obtain up to the linear order

$$\boldsymbol{F}' = \left(T + \frac{1}{2}EI\kappa_0^2\right)'\boldsymbol{\tau_0} + \left(\kappa_0T - EI\kappa_0''\right)\boldsymbol{n_0} + \left(\left(T - EI\kappa_0^2\right)Y'' - \frac{3}{2}EI\left(\kappa_0^2\right)'Y' - EIY^{(4)}\right)\boldsymbol{e_y}.$$
(18)

#### A.3. Linearization of the fluid forces

The oscillating lift force  $p_w$  defined in (3) is a linear order term by assumption. On the other hand, the resistive and reactive forces (1), (2) both include a leading order and a linear order term. First, the expansion of the relative velocity up to the linear order in Y gives  $U_{rel} = \dot{Y}e_y - Ue_x = \dot{Y}e_y - US_0\tau_0 - UC_0n_0$  so that projection on the tangent direction and its orthogonal plane using (16) yields

$$U_{\tau}\boldsymbol{\tau} = -US_{0}\boldsymbol{\tau}_{0} - US_{0}Y'\boldsymbol{e}_{y}, \qquad U_{N}\boldsymbol{N} = -UC_{0}\boldsymbol{n}_{0} + \left(\dot{Y} + US_{0}Y'\right)\boldsymbol{e}_{y}.$$
(19)

Making use of that decomposition in (1) and (2), we obtain the linearized fluid loads

$$\boldsymbol{p_{am}} = -m_a U^2 \left( S_0^2 - \frac{1}{2} C_0^2 \right) \kappa_0 \boldsymbol{n_0} - m_a \left( \ddot{Y} + 2U S_0 \dot{Y}' + U C_0 \kappa_0 Y' + U^2 \left( \left[ S_0^2 + \frac{1}{2} C_0^2 \right] Y' \right)' \right) \boldsymbol{e_y}$$
(20)

$$\boldsymbol{p}_{\boldsymbol{d}} = \frac{1}{2} \rho C_D D U^2 |C_0| C_0 \boldsymbol{n}_{\boldsymbol{0}} - \frac{1}{2} \rho C_D D U |C_0| \left( \dot{Y} + U S_0 Y' \right) \boldsymbol{e}_{\boldsymbol{y}}.$$
(21)

#### A.4. Governing equations

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Finally, the linearized structural acceleration reads  $\ddot{r} = \ddot{Y} e_y$  so that substitution of (18), (20), (21) and (3) in the first Kirchhoff equation (11) provides, after projection along  $\tau_0$  and  $n_0$ , the leading order system of equation

$$\left(T + \frac{1}{2}EI\kappa_0^2\right) = 0, \qquad \left(\kappa_0 T - EI\kappa_0''\right) + \frac{1}{2}\rho C_D DU^2 |C_0| C_0 - m_a U^2 \left(S_0^2 - \frac{1}{2}C_0^2\right)\kappa_0 = 0$$
(22)

with leading order boundary conditions  $\theta_0 = 0$  at s = 0 and  $T = \kappa_0 = \kappa'_0 = 0$  at s = L. The first equation provides the expression of the tension  $T = -1/2EI\kappa_0^2$ . Replacing *T* in the second equation yields the static equilibrium equation (6) after non-dimensionalization. Similarly, projection of (11) on  $e_y$  yields

$$\begin{split} m\ddot{Y} &= \left( \left( T - EI\kappa_0^2 \right) Y'' - \frac{3}{2} EI(\kappa_0^2)'Y' - EIY^{(4)} \right) \\ &- m_a \left( \ddot{Y} + 2US_0 \dot{Y}' + UC_0 \kappa_0 Y' + U^2 \left( \left[ S_0^2 + \frac{1}{2} C_0^2 \right] Y' \right)' \right) \\ &- \frac{1}{2} \rho C_D DU |C_0| \left( \dot{Y} + US_0 Y' \right) + \frac{1}{4} \rho C_L^0 DU^2 C_0^2 q \end{split}$$
(23)

which gives the VIV equation (7) after replacement of the tension and non-dimensionalization.

#### Appendix B. Validity of the linearization

The model used throughout this study is based on the assumption that the only relevant source of nonlinearity in the dynamics comes from the damping term in the Van der Pol wake oscillator. This term is indeed sufficient to ensure the saturation of the vibration amplitude in the limit-cycle to a magnitude consistent with physical observations. However, the linearization leading to the governing equation for the structure (7) relies on the assumption that the transverse displacement Y is a small perturbation to the leading order deformation  $\mathbf{r}_0(s)$ , while the amplitude of the limit-cycle oscillations due to VIVs is not infinitesimal. Thus, the adequacy of the linearization detailed in Appendix A for modelling VIVs requires verification.

The linearization of the structural model performed in Appendix A.2 is purely geometrical and holds as long as  $Y \ll L$ . The numerical results displayed on Figs. 7(a) and 9(a) indicate that the amplitude of the transverse perturbation does not exceed  $Y \leq 0.5D$  in the case of VIVs. The geometrical linearization thus still holds in the limit-cycle for the slender structures considered in this work  $\Lambda = L/D \gg 1$ .

On the other hand, the linearization of the fluid forces performed in Appendix A.3 relies on the additional assumption that the transverse velocity is small compared to the free-stream  $\dot{Y} \ll U$ . The frequency of the VIVs is approximately equal to  $f_w^0 = S_t U/D$  as shown on Fig. 3. Assuming an amplitude  $Y \sim 0.5D$  and with  $S_t = 0.2$ , the order of magnitude of the transverse velocity scales as  $\dot{Y} \sim 2\pi f_w^0 Y \sim 0.6U$ , which is at the mathematical limit of validity of the linearization. Nonetheless, considering only the linear contribution of the fluid forces in the dynamic equation is consistent with the leading-order nature of the models generally used in VIV studies involving wake oscillator models (Skop and Luo, 2001; Facchinetti et al., 2004b; Mathelin and de Langre, 2005; Mukundan et al., 2009). A more detailed discussion about the appropriateness of such an approximation can be found in Skop and Balasubramanian (1997). If the nonlinearity in the Van der Pol wake oscillator is critical in the limitation of the amplitude of the vibrations in the limit-cycle, we expect that the influence of other nonlinearities should be less significant on a qualitative point of view. More details about the influence of nonlinearities arising from the structure, the fluid forces, or the coupling between them can be found in Srinil and Zanganeh (2012).

Finally, when the system becomes unstable to flutter, the amplitude of the oscillations becomes too large for the geometrical linearization of Appendix A.2 to remain valid. In this case, nonlinear coupling between the transverse and inplane directions in the structural equation would eventually lead to a complex large-amplitude 3D motion.

#### Appendix C. Linear stability analysis

To perform the linear stability analysis of the coupled system (7)–(8), we neglect the nonlinear term in Eq. (8) and assume a perturbation of the form  $(Y, q)(s, t) = (\phi(s), \psi(s)) e^{i\omega t}$ . The coupled system (7)–(8) then yields

$$-u^{2}\omega^{2}\phi + i\omega\frac{\beta u^{2}}{S_{t}\Lambda}\left[2S_{0}\phi' + C_{0}\left(\kappa_{0} + c_{d}\Lambda\right)\phi\right] + \left(\left[\frac{\beta u^{2}}{S_{t}^{2}\Lambda^{2}}\left(S_{0}^{2} + \frac{1}{2}C_{0}^{2}\right) + \frac{3}{2}\kappa_{0}^{2}\right]\phi'\right)' + c_{d}\frac{\beta u^{2}}{S_{t}^{2}\Lambda}C_{0}S_{0}\phi' + \phi^{(4)} - c_{l}\frac{\beta u^{2}}{S_{t}^{2}}C_{0}^{2}\psi = 0$$
(24)

$$-\omega^{2}(\psi - A\phi) - i\omega\varepsilon (2\pi C_{0})\psi + (2\pi C_{0})^{2}\psi = 0$$
(25)

with boundary conditions  $\phi = \phi' = 0$  at s = 0 and  $\phi'' = \phi''' = 0$  at s = 1. We then solve the coupled quadratic eigenvalue problem (24)–(25) at the discretization points with the MatLab function quadeig of Hammarling et al. (2013). The linear frequency  $f_{lin}$  and growth rate  $\sigma$  of the linear modes are then related to the eigenvalues thus found by  $\omega = 2\pi f_{lin} - i\sigma$ .

#### Appendix D. Structural stability thresholds

The destabilization of the structural modes is due to the influence of the reactive force in the structural equation (7), and the effect of the oscillatory lift force due to the wake is most likely negligible in this regard. To compute the stability threshold for the structural modes only, we may thus neglect the coupling between Eqs. (7)–(8) and consider exclusively the left-hand side of (7). Besides, the frequency of shedding  $f_w^0$  is irrelevant in this case and we rescale the dimensional time with respect to the characteristic structural time  $T_s = L^2 \sqrt{(m + m_a)/EI}$  instead of  $f_w^0$ . The governing equation then reduces to

$$\ddot{Y} + v\sqrt{\beta} \left[ 2S_0 \dot{Y}' + C_0 \left(\kappa_0 + c_d \Lambda\right) \dot{Y} \right] + \left( \left[ v^2 \left( S_0^2 + \frac{1}{2} C_0^2 \right) + \frac{3}{2} \kappa_0^2 \right] Y' \right)' + c_d \Lambda v^2 C_0 S_0 Y' + Y^{(4)} = 0$$
(26)

where the flutter-specific reduced velocity  $v = UL\sqrt{m_a/El}$  is related to the VIV-specific reduced velocity used throughout this article through  $u = (S_t/\sqrt{\beta})Av$ , and to the Cauchy number through  $C_Y = c_d Av^2$ . The stability thresholds found by linear stability analysis of Eq. (26) for  $\beta = 0.5$  and three different aspect ratios are provided in Table 2 in terms of the three parameters v, u and  $C_Y$ .

The threshold for  $\Lambda = 10$  in terms of the VIV-specific reduced velocity  $u_c = 35.8$  is very close to that found in Section 3.3 for the coupled system (7)–(8),  $u_c = 34.9$ . This confirms the very limited influence of the wake coupling on the structural stability. Besides, the thresholds expressed in term of the flutter-specific reduced velocity v seems almost insensitive to the aspect ratio  $\Lambda$ . Consequently, according to the scalings of the VIV-specific reduced velocity  $u \propto \Lambda v$  and Cauchy number  $C_Y \propto \Lambda v^2$  provided here, the stability thresholds in terms of both these parameters is close to proportional to the aspect ratio  $\Lambda$ .

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