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Adding streamwise streaks in the plane Poiseuille flow

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Abstract

In recent investigations, finite amplitude streamwise streaks, generated with roughness elements, have been used to delay transition to turbulence. The maximum height of these roughness elements is limited by the appearance of instabilities in their near wake, therefore putting a limit on the maximum streak amplitude they can produce. Here we prove that large amplitude streaks can be generated by 'adding' lower amplitude streaks with multiple arrays of roughness elements. ***To cite this article: M. Hollands, C. Cossu, C. R. Mecanique 337 (2009).***

Résumé

Additionner des streaks dans l'écoulement de Poiseuille plan. Dans des études récentes, des *streamwise streaks*, engendrés avec des éléments de rugosité, ont été utilisés pour retarder la transition à la turbulence. La hauteur maximale de ces éléments de rugosité est limitée par l'apparition d'instabilité dans leur sillage proche, ce qui limite l'amplitude maximale des *streaks* qu'ils peuvent produire. Ici nous montrons que des *streaks* de grande amplitude peuvent être engendrés en « additionnant » des *streaks* d'amplitude inférieure par l'utilisation de lignes multiples d'éléments de rugosité. ***Pour citer cet article : M. Hollands, C. Cossu, C. R. Mecanique 337 (2009).***

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Mots-clés : Mécanique des fluides ; Streaks ; Contrôle d'écoulements

1. Introduction

The scope of the present investigation is to determine if streamwise streaks can be 'added' to each other to produce larger amplitude streaks. The streamwise streaks are spanwise modulations of the streamwise velocity that can be efficiently induced in shear flows by initial streamwise vortices through the lift-up effect [1–3]. The initial energy of the vortices can be largely amplified through this effect. The growth of the streaks is transient in nature, it is related to the

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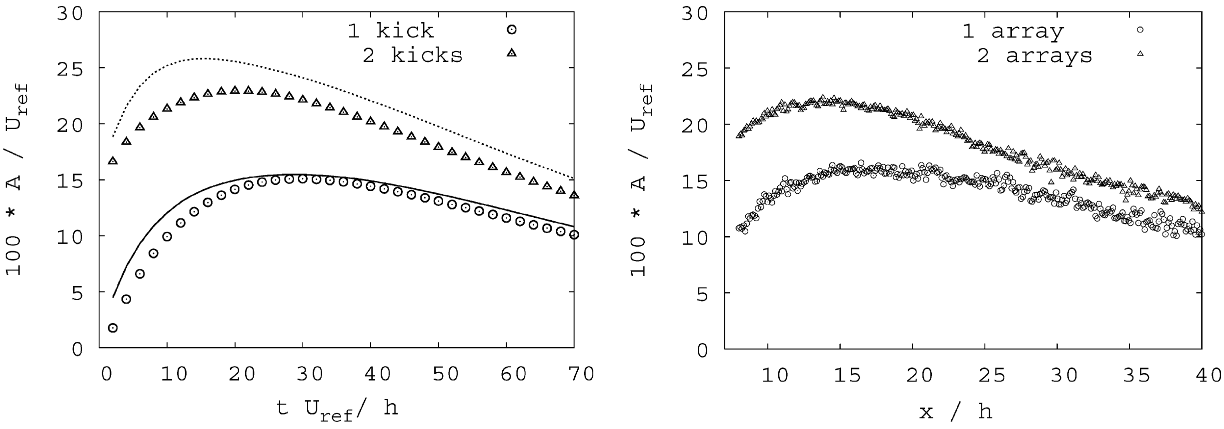
non-normality of the linearized Navier–Stokes operator and can be maximized using optimal perturbations [4]. If their amplitude exceeds a, typically large, critical value, the streaks undergo secondary inflectional instabilities. However, it has been recently shown that artificially forced streamwise streaks of moderate amplitude can have a *stabilizing* action on Tollmien–Schlichting waves in the Blasius boundary layer [5,6]. A series of experiments [7,8] verified this stabilizing effect in the wind tunnel and proved that these streaks can effectively delay transition [9]. In these investigations a spanwise array of evenly spaced roughness elements was used to force the streamwise vortices that induce the streaks. The energy of the initial vortices, and therefore the amplitude of the forced streaks, was increased by increasing the height k of the roughness elements. However, varying too much the height k resulted in undesired modifications of the shape of the forced vortices and streaks and above a critical height (a critical value of the roughness-element Reynolds number) a vortex shedding instability developed in the near-wake of the roughness elements [8]. This has limited the maximum streak amplitudes attained in these experiments and therefore their stabilizing action. Another problem is that, as the growth of the streaks is transient, their stabilizing action is felt only on a finite distance in the downstream direction. A possible solution to these problems could be to use multiple arrays of roughness elements to try to 'add' the streaks they induce. This kind of solution could allow to increase the maximum attainable streak amplitude with a given type of roughness element and/or enlarge the downstream region where the streaks have a desired amplitude. Such a solution does not seem to have been already studied for the steady nearly-optimal type of streaks we are interested in. In particular, it is not at all clear that streaks can be effectively 'added' in such a way. Any additional downstream array of roughness elements is immersed in an already streaky flow with strong wall-normal vorticity. This wall-normal vorticity is maximum right at wall-normal position where the additional vorticity is generated by the roughness elements and this may have a negative influence on this generation process. We have therefore used numerical simulations and experiments to test the 'additivity' of the streak generation process. We choose as a test case the plane Poiseuille flow, which has the advantage to rule out non-parallel effects from the picture.

2. Numerical and experimental results

We consider the flow of an incompressible viscous fluid of kinematic viscosity ν in a plane channel of half height h. We denote by x, y and z the streamwise, wall-normal and spanwise coordinates respectively, with $y/h \in [-1, 1]$. The corresponding velocities are respectively denoted by u, v and w. The Reynolds number $Re = U_{ref} h/\nu$ is defined with respect to the maximum velocity U_{ref} in the laminar flow. The optimal energy growths in the plane Poiseuille flow are obtained with perturbations that are uniform in the streamwise direction and spanwise periodic with an optimal spanwise wavelength $\lambda_z = \pi h$ [10,11]. The energy of the streaks is mostly contained in modulations $\Delta U = U - U_P$ of the streamwise velocity U with respect to the Poiseuille flow solution $U_P(y) = U_{ref}[1 - (y/h)^2]$. Different measures of the amplitude of the induced streaks exist among which the max–min definition [12] given by $A_s(x,t) = [\max_{y,z}(\Delta U) - \min_{y,z} \Delta U]/2U_{ref}$. In the analysis of the experimental results it has proven useful to fit the streamwise velocity at a chosen distance Y from the wall to the function $\widehat{u}(x, Y, z, t) = \bar{u}(x, Y, t) + \widehat{A}(x, Y, t) \sin[2\pi(z - z_0)/\lambda_z]$. The curves $\widehat{A}_s(x, Y, t) = \widehat{A}(x, Y, t)/2U_{ref}$ then give an approximation from below of A_s.

The process of streaks generation with roughness elements is a two-stages process: first streamwise vortices are generated by the roughness elements, then these vortices induce the streaks by the lift-up effect. We first test the additivity idea only on the second phase using direct numerical simulations of the temporal development of streaks from a single or a double 'kick' given by streamwise uniform initial vortices. The optimal vortices are centered in $y = 0$ and are symmetric with respect to that plane [11]. We are however interested in comparisons with the boundary layer experiments and therefore in vortices and streaks generated by roughness elements put on one single wall. While retaining the optimal spanwise wavelength $\lambda_z = \pi h$, we will therefore not give optimal initial vortices as initial condition but the synthetic suboptimal (non-symmetric) velocity field $u_0 = 0$, $v_0 = -A_v[1 - (y/h)^2]e^{-(\frac{y-y_0}{\sigma})^2} \cos(2\pi z/\lambda_z)$ and w_0 derived from the continuity equation. After some preliminary tests, and comparison with preliminary experimental results, the reasonable values $y_0 = -0.36h$ (giving an effective distance of $0.64h$ of the vortices from the bottom wall) and $\sigma = 0.2y_0$ were retained. The Reynolds number is fixed to $Re = 450$, a value large enough to see large transient growths and sufficiently low to avoid too large amplifications of noise and imperfections. The initial amplitude of the vortices $A_v = 0.06$ is selected by requiring A_s not to exceed $0.26U_{ref}$ (the critical value for inflectional instabilities in the Blasius boundary layer [12]). The baseline streaks are computed by direct numerical simulation (see Appendix A) giving the synthetic vortices as initial velocity field at $t_0 = 0$. Both the initial vortices and the ensuing streaks are uniform in the streamwise direction. From Fig. 1a we see that the amplitude of the streaks

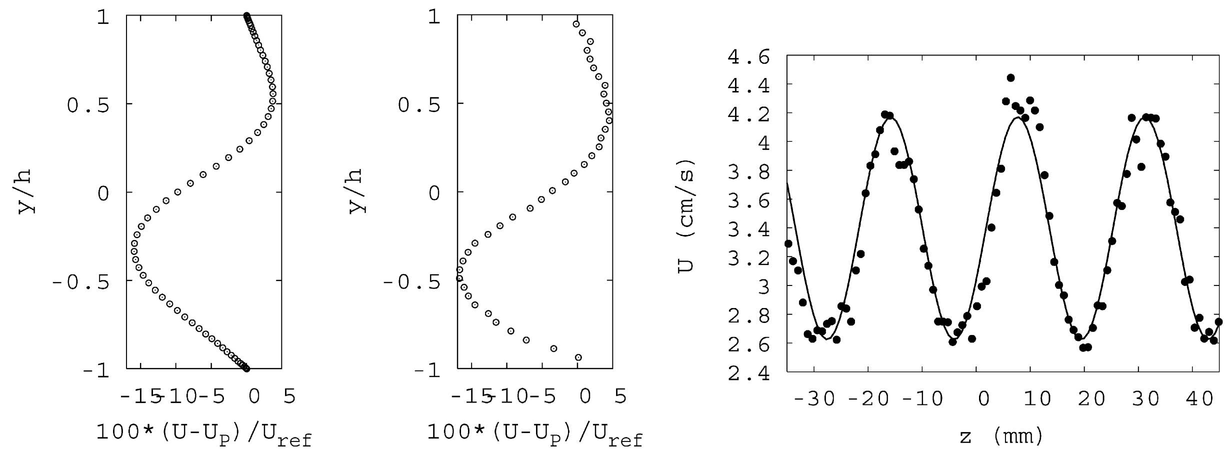


(a) (b)

Fig. 1. Evolution of streaks amplitudes A_s (lines only) and approximated streaks amplitudes $\widehat{A}_S(x, Y = -0.6h)$ (symbols) for respectively single (solid line, circles) and double-kicked (dotted line, triangles) streaks. (a) Results from numerical simulations: evolution of the amplitude in time for streamwise uniform streaks. (b) Results from experiments (only the approximated amplitude is available): evolution of the amplitude in x for steady streaks.

grows in time to reach a maximum amplitude ($A_s \approx 15\%$) at $t_{max} \approx 30h/U_{ref}$. The wall-normal streak profile $\Delta U(y)$ at t_{max} at the spanwise position of the low speed streak is reported in Fig. 2a. We then try to increase the maximum streak amplitude by adding the synthetic initial vortices to the baseline streaks at t_{max}. The numerical simulation of the ensuing 'double-kick' streaks reveals that the idea basically works. From Fig. 1a it is seen that the second kick is able to push the maximum amplitude up to $A_s \approx 25\%$ and that the time interval between the second kick and the maximum amplification is $t^{(2)}_{max} \approx 15h/U_{ref}$. The streaks induced by the second kick are forced on top of streaks that begin to decay at the time of the second kick and, furthermore, it is known that the lift-up amplification is reduced when streaks have larger amplitudes and the maximum amplitude is reached earlier (see e.g. [12,5]). It is therefore not surprising that the maximum amplitude reached with the two kicks is less than the double of the maximum amplitude of the baseline one-kick streaks and that this maximum is reached at an earlier time.

The numerical results prove the efficiency of the lift-up effect in a pre-streaky flow, but the generation of the vortices themselves by the roughness elements has been left unexplored. An experiment has therefore been set-up to investigate the whole process. A Poiseuille flow is first created in a plane water channel enforcing a constant pressure gradient in the streamwise direction; velocity fields are measured with particle image velocimetry (see Appendix A for additional details on the experimental apparatus). The streamwise velocity deviates by less than 3% from the Poiseuille solution in all the measuring area. Baseline streaks have then been generated by placing a single array of roughness elements on the bottom wall at $x_0/h = 100$ and with the optimal spanwise spacing $\lambda_z/h = \pi$. The roughness elements had a square section with a width of $D = \lambda_z/4$ (same ratio as in [7–9]) and a height $k = 0.8h$. The Reynolds number in all the experiments was kept roughly in the range 450 ± 25, depending on the water temperature at the time of the measure. The resulting streaks are steady and experience a spatial growth (in x). A typical wall-normal streak velocity profile $\Delta U(y)$ in the low speed streak z position is reported in Fig. 2b near the station of maximum streak growth x_{max}. The maximum of the low-speed streak amplitude is reached near $y/h = -0.5$. The position of maximum amplitude in the high speed streak is however typically nearer to the wall. Streamwise velocities have therefore been measured in the plane $Y/h = -0.6$. The approximated streaks amplitude curve $\widehat{A}_s(x, Y/h = -0.6)$ (there is no time dependence as the streaks are steady) has then been computed by fitting the velocity profile to the sinusoidal ansatz described above. An example of that ansatz near x_{max} is given in Fig. 2c. The slight scatter on the experimental data points, due to the finite thickness of the laser light sheet and to the strong velocity gradients in the wall-normal direction, does not prevent a reasonable evaluation of the streaks amplitude. The amplitude curve $\widehat{A}_s(x, Y/h = -0.6)$ for the baseline experimental streak, reported in Fig. 1b, is very similar to the curve obtained applying the same postprocessing to the DNS data (reported with symbols in Fig. 1a). From the DNS data we also see that $\widehat{A}_s$ is a reasonable approximation to A_s. A conversion velocity of $\approx 0.6U_{ref}$ can be used to qualitatively compare the temporal evolution (DNS) reported in Fig. 1a to the spatial (experimental) evolution reported in Fig. 1b. A second



(a) (b) (c)

Fig. 2. Typical streaky velocity profiles. (a) Numerical velocity profile $\Delta U(y)$ in the z position of the low speed at the time of maximum streak amplitude. (b) Same profile for the experimental streaks near the position of maximum streak amplitude x_{max}. (c) Example of experimental $U(x = x_{max}, Y = -0.6h, z)$ velocity profile (dots) and its best sinusoidal fit (line).

identical array of roughness elements has then been added at a distance x_{max} from the primary array so as to generate the double-kicked streaks whose amplitude is reported in Fig. 1b. This curve also compares reasonably well with the one obtained applying the same postprocessing to the double-kicked streaks obtained from DNS. The DNS results also suggest that the underestimation of the amplitude given by $\widehat{A}_s(x, Y = -0.6h)$ increases for the double-kicked streaks. The ratio of the maximum amplitude obtained with double and single-kicks is also seen to be slightly reduced in the experiments compared to the DNS. This means that the generation of the vortices from the roughness elements is probably slightly less efficient in the presence of the primary streaks. Very similar results (not shown) are found repeating the experiments with roughness elements of circular cross-section.

3. Conclusion

We have proved that it is possible to 'add' streaks through a multi-stage generation process in order to obtain streaks of larger amplitude. The concept has been first tested numerically in a temporal direct numerical simulation of the development of streaks from streamwise vortices in the plane Poiseuille flow. Adding to the primary streaks, when they reach their maximum amplitude, additional vortices results in larger amplitude streaks. The concept has been then tested experimentally in a water channel by using arrays of roughness elements to generate the vortices, that then induce the streaks by the lift-up effect. Adding a second array of roughness elements at the streamwise position where the primary streaks have maximum amplitude results in larger amplitude secondary streaks. Despite the differences between the two tests (numerical vs. experimental, temporal growth vs. spatial growth, slightly different wall-normal profiles of the streaks, synthetic initial vortices vs. real vortices generated by the roughness elements, etc.) the results are strikingly similar, therefore confirming the robustness of the protocol. These results open the way to the study of similar solutions in other flows, such as the flat plate boundary layer. In that case, one could imagine to add streaks either to increase their maximum amplitude or to extend downstream the region of given amplitude and thus extend their stabilizing action. Of course additional experiments are necessary to test this idea and in particular to rule out possible destabilizing interactions between the additional arrays of roughness elements and the Tollmien–Schlichting waves. Other interesting open questions concern the existence of optimal roughness distributions and their relation to bio-inspired rough skins in laminar flows [13].

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Appendix A. Methods

The numerical simulations are performed with the *cha* code, described in Ref. [14] to which we refer the reader for details. The three-dimensional, time dependent, incompressible Navier–Stokes equations are solved using a Fourier representation in the streamwise and spanwise directions and Chebyshev polynomials in the wall-normal direction, together with a pseudo-spectral treatment of the nonlinear terms. The time integration is based on a four-step low-storage third-order Runge–Kutta method for the nonlinear terms and a second-order Crank–Nicolson method for the linear terms. This code, extensively tested (see e.g. [14,5]). In the present investigation $4 \times 65 \times 32$ points have been used in respectively the x, y and z directions in a computational box of size $0.3h \times 2h \times \pi h$. The domain is extremely short in the x direction as the simulated flow is invariant in that direction.

The experiments have been conducted in a water channel maintained at constant pressure gradient with the use of an inflow and an outflow reservoirs with constantly active overflow walls of controlled height. The channel was closed circuit with a water supply guaranteed by a centrifugal water pump. The channel has a rectangular cross section built with glass plates. The roughness elements are mounted on a removable plexiglas flat plate that is fixed on the bottom of the test section. The test section length is $L = 150$ cm, its width is $B = 30$ cm and its height is $2h = 1.5$ cm when the additional plexiglas plate is mounted. A convergent section with 10:1 ratio and a series of honeycomb filters have been installed before the test section entrance to attenuate the inflow perturbations and to allow for an already developed Poiseuille flow at the test section entrance. The velocity fields have been measured using a Davis particle image velocimetry (PIV) system with a 3W Yag laser and a 1600×1200 pixels ImagerPro2XM CCD camera. The thickness of the planar laser sheet was less than 1 mm and the measured velocities were typically in the range 2–5 cm/s. The laser was used at 50% to 80% of its maximum power and was triggered with double pulses separated by $\Delta t = 9000$ µs. To increase the quality of the measures, 80 to 150 velocity fields acquired at 4 to 8 Hz and derived from correlations on an interrogation windows of 16×16 or 32×32 pixels, have been averaged for each measure. For the measures in the $Y = const$ planes, the camera field was of 6.75 cm in x and 9 cm in z and four displacements of the camera in the streamwise direction were necessary to cover the whole field extending from 6 cm to 30 cm downstream of the roughness elements.

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