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LOCAL AND GLOBAL INSTABILITY OF FLUID-CONVEYING PIPES ON ELASTIC FOUNDATIONS

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We investigate the relationship between the local and global bending motions of fluid-conveying pipes on an elastic foundation. The local approach refers to an infinite pipe without taking into account its finite ends, while in the global approach we consider a pipe of finite length with a given set of boundary conditions. Several kinds of propagating disturbances are identified from the dispersion relation, namely evanescent, neutral and unstable waves. As the length of the pipe is increased, the global criterion for instability is found to coincide with local neutrality, whereby a local harmonic forcing only generates neutral waves. For sets of boundary conditions that give rise only to static instabilities, the criterion for global instability of the long pipe is that static neutral waves exist. Conversely, for sets of boundary conditions that allow dynamic instabilities, the criterion for global instability of the long pipe corresponds to that for the existence of neutral waves of finite nonzero frequency. These results are discussed in relation with the work of Kulikovskii and other similar approaches in hydrodynamic stability theory.
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1. INTRODUCTION

In this paper, the influence of the local properties of bending waves on the global linear stability of a long fluid-conveying pipe with various boundary conditions is analysed. A schematic view of a cantilevered pipe on an elastic foundation is given in Figure 1. In the literature, considerable attention has been given to the lateral vibrations of pipes containing or surrounded by a moving fluid. Many studies [see Païdoussis (1998)] have sought to establish the instability conditions for fluid-conveying pipes of finite length in various geometrical and physical configurations. It has been demonstrated that these pipes can be destabilized by flutter or buckling, depending on the boundary conditions at the ends, as the fluid velocity is increased. Such instabilities will be referred to as *global*.

It has also been shown (Roth 1964; Stein & Tobriner 1970) that an infinite pipe modelled with the same local equations can experience instability. In that case the instabilities are referred to as *local*, since only the local equations are considered, independently of end conditions. Recently, the concepts of absolute and convective instability have been applied to this particular problem (Kulikovskii & Shikina 1988; Triantafyllou 1992; de Langre & Ouvrard 1999). These concepts, first introduced in plasma physics (Briggs 1964; Bers 1983) and fruitfully applied to hydrodynamics (Huerre & Rossi 1998) pertain to the long-time impulse response of a spatially homogeneous system of infinite extent.

The objective of the present paper is to determine the respective roles of wave propagation and wave reflections at the pipe ends on the global instability as the length takes large values, and thus to establish a link between local and global analyses. Such an approach has already been applied to the case of the Ginzburg–Landau equation, a simple substitute for

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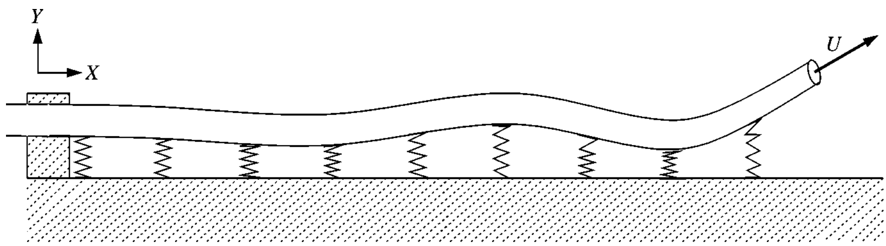

Figure 1. The cantilevered fluid-conveying pipe on elastic foundation.

the Navier–Stokes equations, and the key role of absolute instability has been brought out (Chomaz & Couairon 1999). In the case of fluid-conveying pipes (Benjamin 1960; Païdoussis 1970), the energy exchange rate between the fluid and the structure over one period of oscillation has been expressed in terms of the motion of the pipe ends. Lucey (1998) and Wiplier & Ehrenstein (2000) explored numerically the effect of finite length on the instability of a flexible panel under flow. In Lee & Mote (1997), wave reflections at the boundaries were also shown to result in an energy gain that could lead to global instability. More recently, Inada & Hayama (2000) analysed the role of travelling waves in flutter instability caused by leakage flow. In a more general case, it has been shown by Kulikovskii (1966) that, as the length of a system is increased, the criterion for global stability tends asymptotically to a form that is independent of the boundary conditions.

The linearized equation of motion governing the lateral in-plane deflection $Y(X,T)$ of a fluid-conveying pipe is (Bourrières 1939; Païdoussis 1998)

$$EI\frac{\partial^4 Y}{\partial X^4}+(\rho A U^2)\frac{\partial^2 Y}{\partial X^2}+(2\rho A U)\frac{\partial^2 Y}{\partial X\partial T}+(m+\rho A)\frac{\partial^2 Y}{\partial T^2}+SY=F(X,T), \quad (1)$$

where EI is the flexural rigidity of the pipe, ρA the fluid mass per unit length, U the plug flow velocity, S the elastic foundation modulus and $F(X,T)$ is the external force per unit length. We only consider here the onset of instabilities, and nonlinear effects are therefore neglected in the dynamics of the pipe. If one considers a pipe of length L, appropriate nondimensional variables are, following Gregory & Païdoussis (1966),

$$x=X/L, \qquad y=Y/L, \qquad t=(EI/(\rho A+m))^{1/2}T/L^2,$$

$$u=UL(\rho A/EI)^{1/2}, \qquad \beta=\rho A/(\rho A+m), \qquad s=SL^4/EI, \qquad f=FL^3/EI. \quad (2)$$

Equation (1) then reads

$$\frac{\partial^4 y}{\partial x^4}+u^2\frac{\partial^2 y}{\partial x^2}+2\sqrt{\beta}u\frac{\partial^2 y}{\partial x\partial t}+\frac{\partial^2 y}{\partial t^2}+sy=f(x,t). \quad (3)$$

In the following sections, we consider various boundary conditions at the pipe ends, $x=0$ and $x=1$: $y=\partial y/\partial x=0$ (clamped end), $y=\partial^2 y/\partial x^2=0$ (pinned end) or $\partial^2 y/\partial x^2=\partial^3 y/\partial x^3=0$ (free end).

In Section 2, we give the global conditions of stability for various boundary conditions. The stability conditions of the infinite pipe and the properties of propagating waves are analysed in Section 3. In Section 4, a comparison between the local and global approaches is made regarding the predicted behaviour of the pipe as its length is increased. The results are discussed in Section 5.

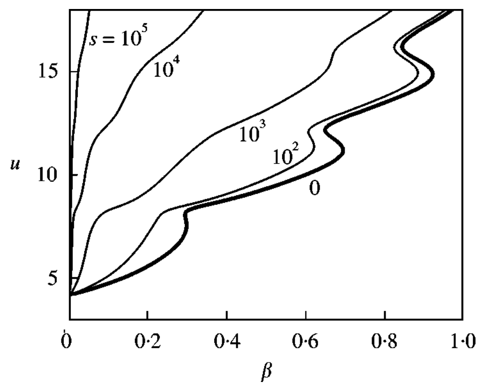

Figure 2. Global stability curves of the clamped–free pipe in the (β, u) plane for different values of the elastic foundation stiffness s.

2. GLOBAL STABILITY

Let us first analyse the linear stability of a fluid-conveying pipe of finite length on an elastic foundation. Three different sets of boundary conditions are considered at the upstream and downstream ends: clamped–free (cantilevered pipe), pinned–pinned and clamped–clamped. Free motion of the pipe is assumed, so that $f(x, t) = 0$ in equation (3).

Following Lottati & Kornecki (1986), the dynamics of the cantilevered (clamped–free) pipe is analysed by calculating its eigenfrequencies $\omega_j, j = 1, \ldots, N$, via a standard Galerkin method (Gregory & Païdoussis 1966), $y(x, t) = \sum_{j=1}^{N} \phi_j(x) e^{-i\omega_j t}$, $\phi_j(x)$ being the jth eigenmode of the pipe without flow and elastic foundation ($u = 0$, $s = 0$). These frequencies will be referred to as global. The real part of ω is the dimensionless oscillation frequency, while its imaginary part is the temporal growth rate. For $u = 0$, the system described by equation (3) is a neutrally stable beam with real eigenfrequencies. By increasing u, some eigenfrequencies become complex and a positive imaginary part for one of them induces a flutter-type instability. The critical nondimensional velocity u_c for the onset of instability is plotted in Figure 2 as a function of β for several values of the elastic foundation stiffness s. Up to 100 modes have been used at the highest values of u and s. The value of u_c clearly depends on β, and the elastic foundation modulus s has a stabilizing effect, as noted in Lottati & Kornecki (1986).

The stability conditions of the pinned–pinned and clamped–clamped pipes on an elastic foundation have been obtained by Roth (1964) and can be derived from that of a column under compressive load (Timoshenko & Gere 1961). The critical velocity for the pinned–pinned pipe is given by

$$u_c = N\pi \left(1 + \frac{s}{(N\pi)^4}\right)^{1/2}, \tag{4}$$

where N is the smallest integer satisfying $N^2(N + 1)^2 \geq s/\pi^4$. The critical velocity at $s = 0$ is readily found as $u_c = \pi$. Similarly, the clamped–clamped pipe undergoes an instability at a critical velocity

$$u_c = 2\pi \left(1 + \frac{3s}{16\pi^4}\right)^{1/2} \tag{5}$$

for $s \leq (84/11)\pi^4$, and

$$u_c = \pi \left(\frac{N^4 + 6N^2 + 1}{N^2 + 1} + \frac{s}{\pi^4(N^2 + 1)}\right)^{1/2} \tag{6}$$

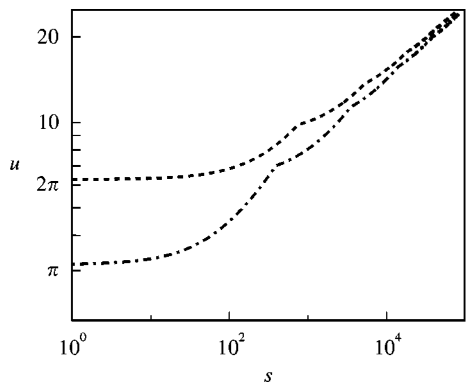

Figure 3. Global stability curves of the pinned–pinned pipe and the clamped–clamped pipe; - - -, clamped–clamped pipe; - · -, pinned–pinned pipe.

otherwise, where N is the smallest integer satisfying $N^4 + 2N^3 + 3N^2 + 2N + 6 \geq s/\pi^4$. The pinned–pinned and clamped–clamped pipes are unstable due to buckling and the critical velocity does not depend on β. The critical velocities for static instability of the pinned–pinned and clamped–clamped pipes are plotted as functions of the foundation stiffness s in Figure 3.

3. LOCAL STABILITY

Consider now an infinite pipe modelled with the same local equation (1) and introduce dimensionless variables that do not refer to the pipe length L but to the local scale $\eta = (EI/S)^{1/4}$, associated with the ratio between bending stiffness and foundation modulus. The length η may be interpreted as the wavelength that appears in the static response of the pipe without flow

$$y(x) = \mathrm{e}^{-x/\eta} \sin\left(2\pi \, \frac{x}{\eta}\right) \tag{7}$$

under transverse unit point loading. Appropriate nondimensional variables are now

$$x = X/\eta, \qquad y = Y/\eta, \qquad t = (S/(\rho A + m))^{1/2} T,$$
$$v = U(\rho A)^{1/2}/(SEI)^{1/4}, \qquad \beta = \rho A/(\rho A + m), \qquad f = F/(S^3 EI)^{1/4}. \tag{8}$$

These will be used throughout the remainder of the present paper. Equation (1) now reads

$$\frac{\partial^4 y}{\partial x^4} + v^2 \frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta} v \frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial t^2} + y = f(x, t). \tag{9}$$

If the pipe displacement is sought in the form

$$y(x, t) = y_0 \mathrm{e}^{\mathrm{i}(kx - \omega t)}, \tag{10}$$

the linear dispersion relation is readily obtained as

$$D(k, \omega, v, \beta)\, y(\omega, k) = [k^4 - v^2 k^2 + 2\sqrt{\beta} v k \omega - \omega^2 + 1]\; y(\omega, k) = \phi(\omega, k), \tag{11}$$

where $\phi(\omega, k)$ is the Fourier transform in x and t of the forcing function $f(x, t)$. Local properties of bending waves propagating along the pipe are now analysed in terms of the wavenumber k and frequency ω through the examination of this dispersion relation.

The system is stable if, for any sinusoidal wave of infinite extent in the x-direction and associated to a real wavenumber k, the corresponding frequencies given by equation (11) are such that the displacement remains finite in time. Stability is therefore ensured if for any real wave number k, $\mathscr{I}m[\omega(k)] \leq 0$. This approach is said to be *temporal*, since it examines the evolution of waves in time. According to Roth (1964) and Stein & Tobriner (1970), the pipe is locally unstable when

$$v > v_i = \sqrt{\frac{2}{1-\beta}}. \tag{12}$$

Conversely, the *spatial* approach refers to the development in space of waves generated by a localized time-harmonic forcing. It is more appropriate for the analysis of the global instability of finite-length systems, since no assumption of infinite extent is made on the perturbation.

Let us first consider the impulse response $G(x,t)$ of the system to an impulsive loading,

$$f(x,t) = \delta(x)\delta(t). \tag{13}$$

If $v < v_i$, the system is stable and the impulse response is an evanescent wavepacket. In the case of instability, we may differentiate between two cases. The absolute or convective nature of the instability is characterized by the long-time impulse response $G(0,t)$ at the impulse point (Bers 1983). If $\lim_{t\to\infty} G(0,t) = 0$, the instability is said to be convective. The amplified wavepacket created by the impulse forcing is convected downstream of the point of excitation. If $\lim_{t\to\infty} G(0,\infty) = \infty$ the instability is said to be absolute. The wavepacket grows near $x = 0$ and is dominated by the absolute frequency ω_0 such that

$$\mathscr{I}m(\omega_0) > 0, \quad \text{and} \quad D(k,\omega_0) = \left.\frac{\partial D(k,\omega)}{\partial k}\right|_{\omega_0} = 0. \tag{14}$$

This saddle point (ω_0, k_0) of the dispersion relation must also be associated with a pinching in the complex k plane between two branches that correspond to waves found on either side of the impulse perturbation. This gives a criterion of transition between convective and absolute instability (Bers 1983).

Let us also analyse the properties of waves generated by forcing at a real frequency ω_f. This is done by considering the harmonic forcing

$$f(x,t) = \delta(x)\,H(t)\,\mathrm{e}^{\mathrm{i}\omega_f t}, \tag{15}$$

where $H(t)$ is the Heaviside unit step function. In the case of absolute instability the transient associated with the switching on of the harmonic forcing at $t = 0$, modelled by $H(t)$, contains all the frequencies, including the absolute frequency ω_0 and the system evolution will be dominated by the absolute instability. In the case of convective instability, the switch on transient is advected downstream, and such a forcing generates four waves corresponding to four wavenumber roots of the dispersion relation at $\omega = \omega_f$. Two wavenumbers correspond to waves that propagate downstream of the excitation point and will be referred to as k_{d_1} and k_{d_2}. The two other wavenumbers correspond to waves that propagate upstream of the excitation point and will be referred to as k_{u_1} and k_{u_2}.

In the convectively unstable regime, the direction of each of the waves emerging from the point of excitation may be found by calculating the four roots $k(\omega)$ at a complex frequency ω with $\mathscr{I}m(\omega)$ larger than the maximum growth rate $\tau_{\max}$ (Bers 1983) defined by

$$\tau_{\max} = \max_{k\ real} \mathscr{I}m[\omega(k)]. \tag{16}$$

Complex wavenumbers with a positive imaginary part define waves propagating downstream of the point of excitation, while wavenumbers with a negative imaginary part define waves propagating upstream. By reducing $\mathscr{I}m(\omega)$ to zero and following the evolution of k-branches, we may identify upstream- and downstream-going waves for a real value of ω. A downstream wavenumber with a positive imaginary part corresponds to a spatially evanescent wave and with a negative imaginary part to a spatially amplified wave. A wavenumber that is real refers to a spatially neutral wave. If the system is locally stable, the maximum growth rate as defined in equation (16) equals zero and the direction of propagation for a real forcing frequency may be found directly by considering the imaginary part of k or the group velocity if k and ω are both real.

Depending on the parameter values and the forcing frequency, four possible combinations of waves are found by solving equation (11) for a given real value of ω, as illustrated in Figure 4: Case 1, four evanescent waves; Case 2, two evanescent waves and two neutral waves, one of each downstream, one of each upstream; Case 3, four neutral waves; Case 4, one amplified and one decaying wave travelling downstream, two neutral waves travelling upstream. Figure 5(a) and 5(b) show the evolution of $\mathscr{I}m(k)$ with ω for two typical sets of parameters. When $\beta = 0{\cdot}1$, $v = 1$ [Figure 5(a)], only Cases 1 and 2 are observed. Conversely, with $\beta = 0{\cdot}94$, $v = 7$ [Figure 5(b)], Cases 2–4 are found. For the latter values of β and v, the unforced pipe is convectively unstable.

For a given set of physical parameters β and v, the waves generated by forcing are clearly dependent on the forcing frequency. Assuming that ω explores all real values, we may now analyse the wave-bearing capabilities of the medium in the (β, v) plane.

In the domain of instability, the limit between convective and absolute instability is related to the existence of a triple root of the dispersion relation (Crighton & Oswell 1991; Triantafyllou 1992; de Langre & Ouvrard 1999), which arises at

$$v_{ac} = \left(\frac{12\beta}{8/9 - \beta}\right)^{1/4}. \tag{17}$$

When $v < v_{ac}$, the instability is convective and spatially amplified waves exist in a range of frequencies.

In the domain of stability, $v < v_i$, we may also differentiate between several subdomains in the (β, v) plane. As shown in Figure 6, a set of four neutral waves may appear in two distinct frequency ranges. The first range, further referred to as "static range", is bounded by $\omega = 0$ and ω_1, while the second, further referred to as "dynamic range" is bounded by ω_2 and ω_3. The frequencies ω_1, ω_2 and ω_3 are associated with wavenumbers that are double roots of the dispersion relation. Therefore, the emergence of the static range $[0, \omega_1]$ may be related to the existence of a double wavenumber root at $\omega = 0$ for a critical value v_s of the reduced velocity. We then have

$$D(k, 0, \beta, v_s) = \left.\frac{\partial D(k, \omega, \beta, v_s)}{\partial k}\right|_{\omega = 0} = 0, \tag{18}$$

which yields

$$v_s = \sqrt{2}. \tag{19}$$

Thus, if $v > v_s$ there exists a range $[0, \omega_1]$ in which all the waves are neutral. At the emergence of the dynamic range, the two double roots at ω_2 and ω_3 are identical (see Figure 6), forming a triple root of the dispersion relation. This second threshold for the appearance of a dynamic range is therefore also given by equation (17), but extended in the

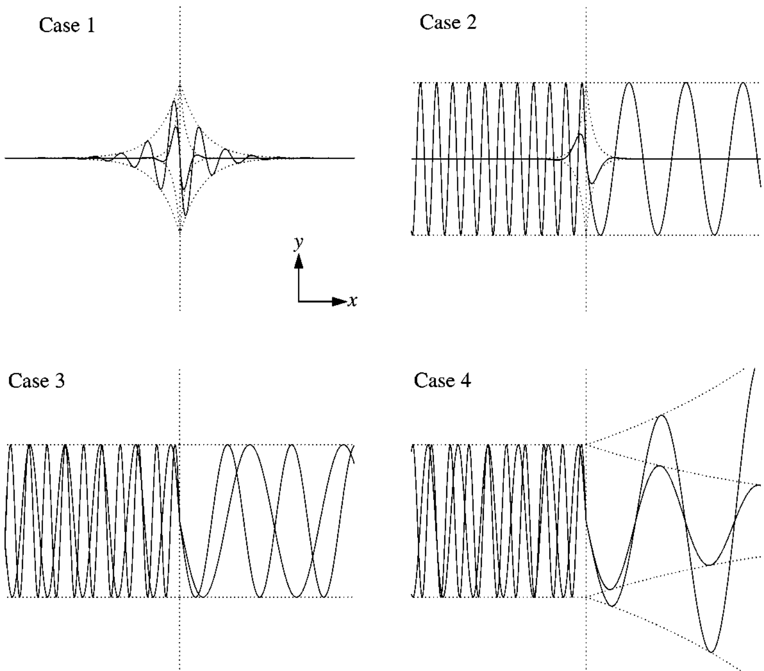

Figure 4. Schematic view of waves generated by the harmonic forcing: Case 1, evanescent waves; Case 2, neutral and evanescent waves; Case 3, neutral waves; Case 4, neutral, evanescent and amplified waves.

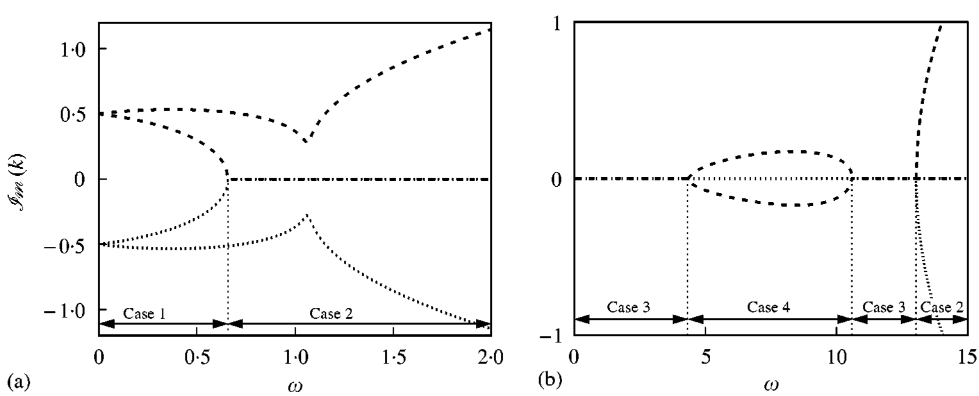

Figure 5. Imaginary part of the wavenumbers as functions of the real frequency: ---, Downstream wavenumbers k_{d_1} and k_{d_2}; $\cdots\cdots$, upstream wavenumbers k_{u_1} and k_{u_2}. (a) $\beta = 0{\cdot}1$, $v = 1$, stability, evanescent and neutral waves only; (b) $\beta = 0{\cdot}94$, $v = 7$, convective instability, evanescent, neutral and amplified waves.

domain of stable parameters. Thus, if $v > v_{ac}$ there exists a dynamic range $[\omega_2, \omega_3]$ in which all the waves are neutral, provided that $v < v_i$ (stability).

Let us summarize the response of the infinite pipe to localized time-harmonic forcing in terms of the parameters β and v.

(a) When

$$v < \sqrt{2} \quad \text{and} \quad v < \left(\frac{12\beta}{8/9 - \beta}\right)^{1/4},$$

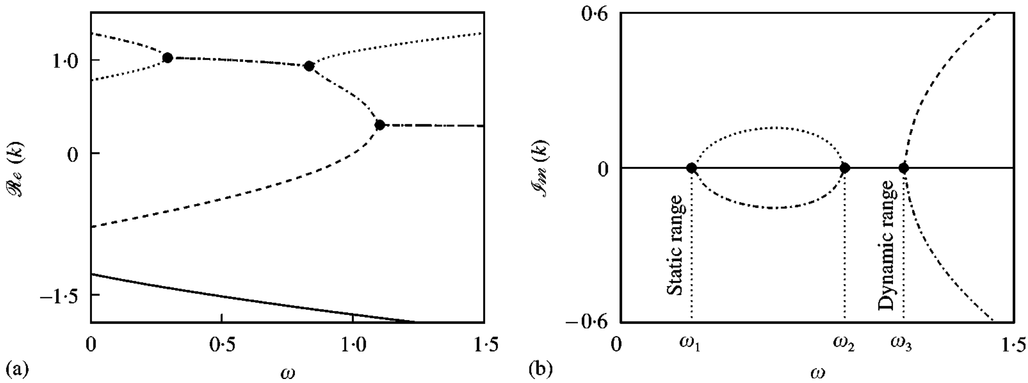

Figure 6. The four roots of the dispersion relation as a function of ω at $\beta = 0{\cdot}15$, $v = 1{\cdot}5$; (a) real part of k, (b) imaginary part of k; ●, location of a second-order root of the dispersion relation. The static and dynamic ranges refer to the values of ω at which there are four neutral waves.

there exist evanescent waves at all forcing frequencies. This domain is further referred to as that of "evanescence" (E).

(b) When

$$v > \sqrt{\frac{2}{1-\beta}} \quad \text{and} \quad v > \left(\frac{12\beta}{8/9-\beta}\right)^{1/4},$$

the response to any forcing is dominated by the complex absolute frequency ω_0 and no wave direction may be defined. This is the domain of "absolute instability" (AI).

(c) When

$$v > \sqrt{\frac{2}{1-\beta}} \quad \text{and} \quad v < \left(\frac{12\beta}{8/9-\beta}\right)^{1/4},$$

there exists a range of forcing frequencies that generates amplified waves. This is the domain of "convective instability" (CI).

(d) In the remaining domain of the (β, u) plane, there exist ranges of forcing frequency where four neutral waves are generated, and no amplified waves exist at other frequencies. If parameters are such that neutral waves are generated by frequencies in the static range as defined above (see Figure 6), the medium is said to be "statically neutral" (SN). In the same manner, a "dynamically neutral" (DN) domain refers to the existence of the dynamic range defined above.

The various domains of the (β, v) plane associated with (a)–(d) are shown in Figure 7.

4. LOCAL NEUTRALITY AS A CRITERION FOR GLOBAL INSTABILITY

We consider again a pipe of finite length L. In nondimensional variables pertaining to wave propagation, as defined in equation (8), the nondimensional length of the pipe is

$$l = L\left(\frac{S}{EI}\right)^{1/4}. \tag{20}$$

In this section we increase the value of l, which is interpreted as increasing the length of the pipe. This is strictly equivalent to increasing the elastic foundation modulus in equation (3) (Doaré & de Langre 2000).

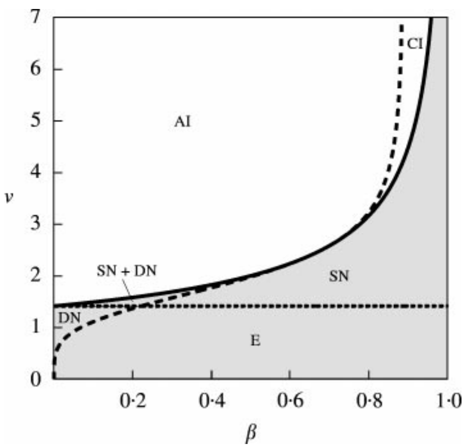

Figure 7. Properties of waves in an infinite pipe: —, local stability criterion, equation (12); ----, criterion for absolute/convective instability transition in the unstable region and for the existence of four neutral waves in the stable region, equation (17); ····-, criterion for the existence of four neutral waves at $\omega = 0$, equation (19). E, evanescent; DN, dynamic neutrality; SN, static neutrality; CI, convective instability; AI, absolute instability; grey region, local stability.

Let us first analyse the behaviour of the pinned–pinned and clamped–clamped pipes. The critical velocities of equations (4)–(6) are first rewritten in the local dimensionless variables of equation (8). For the pinned–pinned pipe, they are

$$v_c = \frac{N\pi}{l}\left[1 + \left(\frac{l}{N\pi}\right)^4\right]^{1/2}, \tag{21}$$

where N is the smallest integer satisfying $N^2(N+1)^2 \geq (l/\pi)^4$, and for the clamped–clamped pipe

$$v_c = \frac{2\pi}{l}\left[1 + \frac{3}{16}\left(\frac{l}{\pi}\right)^4\right]^{1/2} \quad \text{or} \quad v_c = \frac{\pi}{l}\left[\frac{N^4 + 6N^2 + 1 + (l/\pi)^4}{N^2 + 1}\right]^{1/2} \tag{22}$$

for $l^4 \leq (84/11)\pi^4$ or $l^4 \geq (84/11)\pi^4$, respectively, using $N^4 + 2N^3 + 3N^2 + 2N + 6 \geq (l/\pi)^4$. In Figure 8(a), the evolution of the critical velocities is plotted in the (β, v) plane as l is varied. When l is increased to infinity, critical velocities (21) and (22) tend to the same limit, namely

$$v_\infty = \sqrt{2}. \tag{23}$$

As noted in the analysis of buckling under compressive load (Timoshenko & Gere 1961), the critical velocity for global static instability becomes independent of the boundary conditions. The above limit is exactly the lower boundary for static neutrality as sketched in Figure 8(b). Thus, when the pipe length tends to infinity, the global *static* instability arises as soon as neutral waves exist at zero frequency (*static* range) which may be combined into a *static* global mode of infinite extent. Indeed, it was already noted by Païdoussis (1998) that for finite-length pipes with fixed ends, all wavenumbers are real at static instability.

Following a similar approach for the cantilevered pipe, the evolutions of the critical velocity, calculated in Section 2, are now rescaled in terms of local dimensionless variables and plotted in the (β, v) plane as l is varied [Figure 9(a)]. The critical velocity appears to tend to a limit that also falls into the neutral domains of Section 3 (Figure 7) but it differs

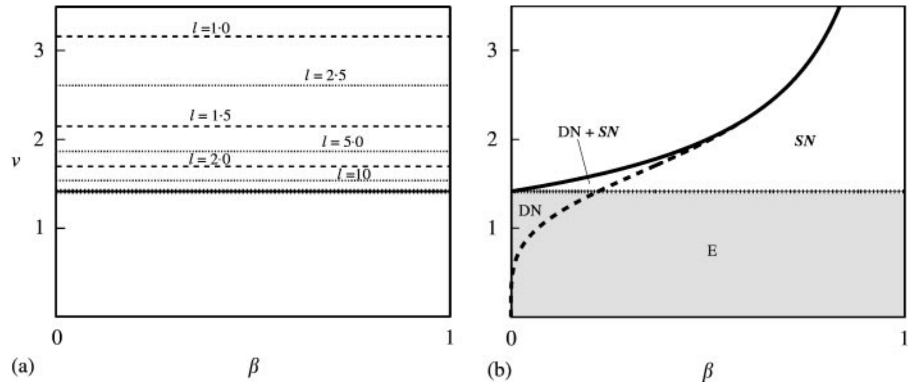

Figure 8. (a) Critical velocities of the pinned–pinned (- - - -) and the clamped–clamped (· · · · ·) pipe for increasing pipe length; the (—) refers to the asymptotic criterion for static instability given by equation (23). (b) Schematic view of the local properties of travelling waves; in the grey region, there exist evanescent waves at $\omega = 0$.

from the limit of the preceding static case. Up to $\beta = 2/3$, this limit appears to coincide with that of dynamic neutrality in Figure 9(b). (Above this limit, a more complex behaviour seems to arise, and much higher values of l should be explored numerically.) Thus, when the length of the pipe tends to infinity, the global *dynamic* instability arises as soon as neutral waves exist at nonzero frequencies (*dynamic* range) that may be combined into a *dynamic* global mode of infinite extent. The fact that only the dynamic range is relevant in this latter criterion is further supported by plotting the critical flutter frequency ω_c at the critical velocity in comparison with the boundaries ω_2 and ω_3 of the dynamic range (Figure 10). Clearly, the global instability sets in the dynamic range of frequencies.

These results may now be analysed following the approach of Kulikovskii, when l tends to infinity. In this framework, all global eigenfrequencies ω_j of the pipe asymptotically satisfy the equation

$$\mathscr{Im}[k_{d_1}(\omega_j) - k_{u_1}(\omega_j)] = 0, \tag{24}$$

where k_{d_1} and k_{u_1} are, respectively, the downstream wavenumber with the smallest imaginary part and the upstream wavenumber with the greatest imaginary part. Depending on the imaginary part of the ω_j, conclusions regarding asymptotic global stability may be directly drawn from equation (24). In our case, three different situations arise in the (β, v) plane. In the domain of “evanescence” (Figure 7), it may be proven by standard branch analysis in the complex ω-plane that no frequency with a positive imaginary part may satisfy equation (24). Hence, stability is ensured. Conversely, in the domains of “instability (convective or absolute)”, equation (24) implicitly defines a curve in the complex ω-plane, with a positive imaginary part. Global asymptotic instability arises. Finally, in the domains of “neutrality”, one may not identify most amplified or less evanescent wavenumbers k_{d_1}, k_{u_1} for all real frequencies, as some range of forcing frequencies generates four neutral waves. Although equation (24) may have some meaning in specific regions of the ω-plane, no general conclusions as to asymptotic global stability may be drawn. Figure 11 summarizes these three cases in the (β, v) plane. Clearly, our results are consistent with the approach of Kulikovskii, as our limit stability cases (Figures 8 and 9), fall into the intermediate domain between the domain of instability and the domain of evanescence of Figure 11. It should be noted that Kulikovskii’s approach implies that boundary conditions have a negligible role at infinite length. It cannot therefore differentiate between our two limit curves (cantilevered and fixed ends).

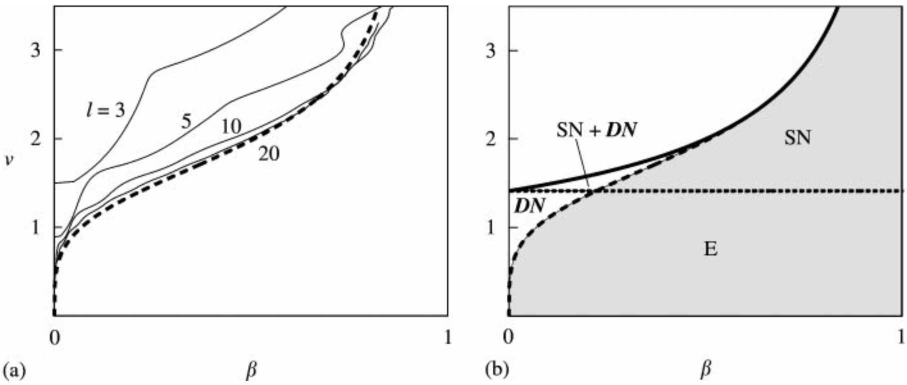

Figure 9. (a) Critical velocities of the cantilevered pipe for increasing pipe lengths (thin lines); the (- - -) line refers to the asymptotic criterion of dynamic instability, equation (23). (b) Schematic view of the local properties of travelling waves; in the grey region, the dynamic range of neutral waves does not exist.

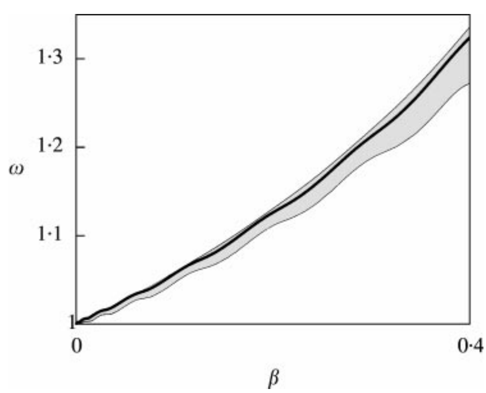

Figure 10. Flutter frequency versus β: —, critical flutter frequency at $l = 20$; shaded domain, dynamic range.

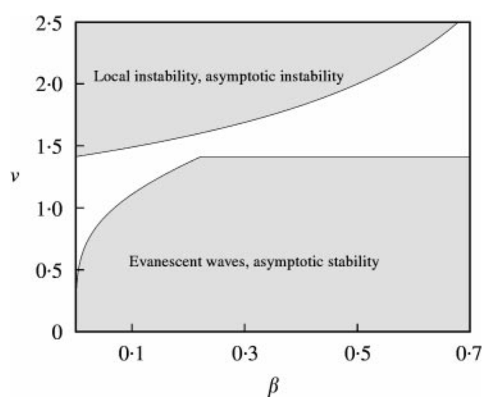

Figure 11. Asymptotic behaviour of the fluid-conveying pipe of infinite length.

5. DISCUSSION

In the analysis of the preceding section, the respective roles of the ends and bulk of the pipe have been found to strongly depend on the ranges of flow velocity v and mass ratio β. A first type of behaviour has been identified, which is fully consistent with the approach of Kulikovskii: in the domains of local evanescence and local instability, when the pipe length tends to infinity, the asymptotic behaviour of the pipe is independent of the boundary conditions at the pipe extremities. This may be understood by considering that evanescent (respectively amplified) waves play an increasingly relative role in the energy balance as the pipe length is increased. Any energy input or output associated with end reflections is ultimately overwhelmed by the stabilizing (respectively destabilizing) effect in the bulk of the pipe. The latter therefore controls global stability. Conversely, in the domain of neutrality there may exist sets of neutral waves that convey energy upstream and downstream without interfering. Even with increasing length, boundary conditions effectively control global stability. In this second type of behaviour, boundary conditions also play a crucial role through the selection of ranges of frequencies. Some end conditions allow flutter to develop [see Païdoussis (1970) and Lee & Mote (1997)], so that energy transfer between flow and pipe may occur in the course of pipe motion. In our approach, for a flutter global mode to actually develop in the domain of dynamic neutrality it is necessary that two conditions be satisfied: (a) the frequency be such that only neutral waves are generated and (b) the corresponding wavenumbers be such that the boundary conditions are satisfied. While the latter condition selects discrete eigenfrequencies, the former requires that one of them falls into the "dynamic range", as defined in Section 4. For infinite pipe length the spectrum of eigenfrequencies becomes continuous and flutter therefore arises as soon as the flow velocity enters the domain of local "dynamic neutrality". The pipe of finite length may yet be stable in this domain, as it may happen that none of its discrete eigenfrequencies fall into the existing "dynamic range" that would allow instability to set in. This is observed in Figure 9, where the stability curve for cantilevered finite pipes is seen to lie above that for the infinite pipe. S-shaped irregularities in the stability curves of finite-length pipes are known to be associated with changes in modal contributions and frequency of instability (Gregory & Païdoussis 1966; Païdoussis 1998). They may also be interpreted as the consequence of discrete frequencies entering or leaving the dynamic range. The same approach may be used when boundary conditions allow only static instability: as the length increases, the spectrum of eigenfrequencies densifies and the occurrence of one of them being equal to zero increases. In Figure 8, the instability threshold is seen to decrease with increasing pipe length. It should be noted that static neutrality may also be defined as marginal local divergence instability (Carpenter & Garrad 1986).

The extension of the present approach to other hydroelastic systems modelled by dispersion relations of the same form (Brazier-Smith & Scott 1984; Crighton 1991; Peake 1997; de Langre & Ouvrard 1999; de Langre 2000) may also be considered. It raises the question of the existence of static or dynamic neutrality in parameter space. In our case, we have shown that dynamic neutrality takes place when the criterion for the existence of a third-order root, equation (17), is satisfied in the domain of local stability. It is an extension into the domain of stability of the criterion for transition between absolute and convective instability (Crighton 1991). Systems such as pipes without foundation (Kulikovskii & Shikina 1988) and plates bounded by uniform flow (Crighton 1991) are locally unstable at the onset of flow. Hence, domains of static or dynamic neutrality may not exist, and it is therefore expected that global instability for long systems arises at zero velocity. In the case of membranes with bounded or unbounded flow (Kelbert & Sazonov 1996; de Langre 2000) where local instability arises at finite flow velocity, the criterion of equation (17) is also found to fall into the domain of instability. Following the present approach, one deduces that no domain of dynamic neutrality may exist. The asymptotic criterion for global instability is probably that of local instability or that of a transition from convective to absolute instability.

We may now draw some comparison with the results of similar approaches in the field of hydrodynamic stability. The Ginzburg–Landau equation is known to be a simple yet fruitful model of a wide class of open flows (Huerre & Monkewitz 1990). It has been shown that for such a model the global criterion for instability tends asymptotically to the local criterion for absolute instability when the length of the medium is increased (Chomaz & Couairon 1999). This conclusion, which differs from that of the present paper, may be understood by taking into consideration the fact that for the Ginzburg–Landau equation no neutral domain exists. Moreover, when convective instability arises, upstream-going waves are strongly evanescent. Any balance involving the amplified downstream-travelling and damped upstream-travelling waves yields a total decay of the energy. The absence of a domain of neutrality implies that boundary conditions play a relatively minor role. The strong evanescence of upstream waves precludes global instability even in the range of local convective instability. These results are fully consistent with those derived from Kulikovskii's approach.

The preceding comparison may be used to shed some light on the very classic problem of destabilization by damping in finite or infinite systems (Païdoussis 1970, 1998; Carpenter & Garrad 1986). If damping terms were to be added in our model of the fluid-conveying pipe the domain of neutrality would vanish. Depending on the value of parameters it would be transfered into the stable or unstable domains. When the length of the pipe is increased (or alternatively the damping coefficient), the criterion for global stability would come closer to the criterion for absolute instability instead of neutrality. This is being further investigated.

6. CONCLUSION

The investigation of the behaviour of the fluid-conveying pipe on an elastic foundation has been carried out as its length is increased. This behaviour has been interpreted in terms of the local properties of the waves in the pipe. In that sense we have defined distinct local configurations pertaining to the properties of the waves generated by a localized harmonic forcing. The range of flow velocity and mass ratio where spatially evanescent waves exist at all real forcing frequencies is said to be that of "evanescence". In the domain of "static neutrality", there exist only neutral waves in a range of forcing frequencies containing the zero frequency. In the domain of "dynamic neutrality", there exist only neutral waves in a range of nonzero frequencies. In the domain of "convective instability", there exist amplified waves at some forcing frequencies. In the domain of "absolute instability", no wave direction can be found at any real frequency and the instability is dominated by the absolute frequency; the response is amplified at the source and gradually contaminates the entire medium. We have thus found that the criterion for global instability as the length is increased becomes closely related to the local properties of the waves in the pipe, but also depends on the boundary conditions. For sets of boundary conditions that allow only static instability, such as pinned–pinned or clamped–clamped ends, we have determined that the asymptotic criterion for global instability is that of static neutrality. Conversely for the cantilevered pipe, which is known to be destabilized by flutter, the asymptotic criterion for global instability is that of dynamic neutrality.

By differentiating the particular effects of the boundary conditions, we have given here a set of results that may not be derived from more classical criteria such as that of Kulikovskii.

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