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Weightlifting and the actomyosin cycle

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How does a human lift a weight? Can we relate the dynamics of the lift to the molecular actin–myosin interactions responsible for muscle contraction? We address these questions with bench press experiments that we analyse with a theoretical model, based on the sliding filament theory. The agreement is fair, and we discuss its possible extension to medical diagnostics.

1. Introduction

The motion of vertebrates results from a coupling between bones-joints and muscles. Even if its study goes back to Aristotle's De motu animalium [1], the first quantitative analysis was produced in 1675 by Giovanni Borelli [2] (figure 1). A review on biomechanics can be found in [3,4]. At this macroscopic scale, the muscle is often characterized by the heuristic Hill's law [5–7] and modelled as an active viscoelastic structure, able to produce a force, to store a tension and to dissipate energy [8].

The precise mechanism of force generation has been itself a long scientific quest since antiquity and the work of Hippocrates of Cos [9,10]. One way to measure the difficulties associated with the understanding of the force generation mechanism consists of comparing the scale at which the force is used (typically the body scale, 1 m) to the scale at which the force is generated which is a topological conformation change of the myosin molecule by 10 nm [11]). Eight orders of magnitude thus separate the molecular origin of the force from its macroscopic function, namely the motion of organisms. Considering the scales involved, this quest has been subjected to the development of techniques from the early microscope (the sarcomere is visualized for the first time in 1674 by van Leeuwenhoek [10]) to the electron



Figure 1. Biomechanical study of holding mass by Borelli in 1675 [2].



Figure 2. (*a*) General principle of bench press exercise. (*b*) Table of morphological characteristics of athletes performing BP experiments: height (*H*), body mass (M_0), arm length (*L*), arm section (*S*), the lightest (M_{min}) and the maximal (M^*) masses they lift during experiments. (Online version in colour.)

microscope [12]. The sliding filament model came out of this quest. According to Needham [9], *the first hints of the sliding-filament mechanism of contraction were given by the low-angle X-ray diffraction patterns obtained by H.E. Huxley with living and glycerol-extracted muscle* in 1953 [13]. A theory for the contraction based on this sliding-filament model was then proposed by A. F. Huxley in 1957 [14] and further completed in 1968 by Deshcherevskii [15]. The connection between the force generated and the precise molecular machinery is still an active field of research [16–18].

The goal of this article is to show how the sliding filament model can be coupled to a classical mechanical joint equation to account for motions observed in weightlifting. The experimental setup is presented in §2, and the results in §3 prior to the model in §4 and the discussion on its possible extension to medical diagnostics in §5.

2. Experimental set-up

In order to minimize the number of joints involved in the motion, we have worked with the bench press configuration presented in figure 2*a*. This is an upper body exercise where the athlete

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Figure 3. Snapshot sequences of the single lifts performed by Gregory at bench press for barbells M = 20 kg (upper sequence) and M = 140 kg (lower sequence). (Online version in colour.)

lies on his back, lowers the barbell to chest level and then pushes it back up until the arms are straight.

The morphological characteristics of the athletes are gathered in the table of figure 2*b*. *H* is their height and M_0 is their mass. The length of their arm, from shoulder to elbow is *L* and *S* is the section of the biceps. Finally, M_{min} is the lighter mass they raised while M^* is their record lift.

3. Experimental results

The athletes are asked to lift the weight once at maximal speed. They perform this explosive task for different masses of barbell from the lightest mass M_{\min} to the maximal mass they are able to lift M^* . Between each lift, a resting period of several minutes is respected.

(a) Qualitative results

The two snapshot sequences of figure 3 present qualitatively the dynamics of the lift. The athlete lifts the barbell from z = 0 (bent arms) to z = 2L (straight arms) in a time T_L . In the upper sequence, the athlete takes 0.4 s to raise a mass of 20 kg. In the lower sequence, it takes him 0.9 s to raise a 140 kg barbell.

(b) Quantitative results

The precise dynamics of the barbell is recorded with an accelerometer Myotest acquiring at 500 Hz which provides velocity and acceleration. Its vertical location is measured using a video camera at 30 fps.

Figure 4 shows the evolution of the vertical position of the barbell $z_M(t)$ and its velocity $\dot{z}_M(t)$ with time, for different masses of the barbell from M_{\min} to a fraction of M^* , and for two different athletes with different morphologies: Michael ($M_0 = 71 \text{ kg}$, $M^* = 105 \text{ kg}$) in figure 4a–c and Gregory

 $(M_0 = 114 \text{ kg}, M^* = 230 \text{ kg})$ in figure 4b-d.

With no surprise, one observes that the dynamics of the barbell is slower for heavier barbells. The position curves show that the height travelled by the barbell decreases a bit when its mass increases. This is due to the holding location of the hands along the bar. The heavier the mass, the larger the distance between the hands and the lower the final height of the barbell.

Concerning the velocities, the barbell is initially at rest and recovers this state at the end of the lift. The initial and final velocities $\dot{z}_M(t=0)$ and $\dot{z}_M(t=T_L)$ are thus null, and one expects a maximum value in between. This is indeed the general shape observed in figure 4*c*–*d*. A further feature is the deformation of the velocity curves when the load increases. For light



Figure 4. Barbell dynamics: vertical position $z_M(t)$ and velocity $\dot{z}_M(t)$ for different masses of the barbell from M_{\min} and M^* and for Michael ($M_0 = 71 \text{ kg}, M^* = 105 \text{ kg}$) on the left (a,c), and Gregory ($M_0 = 114 \text{ kg}, M^* = 230 \text{ kg}$) on the right (b,d). (Online version in colour.)

barbell, the velocity signal is rather symmetric, whereas for heavier barbell, a second bump systematically appears.

The dynamics of the barbell is presented for two athletes with different morphologies: one observes that the general trends are similar, but a careful analysis of the curves reveals some differences, particularly in the shape of the velocity curves. For heavy weights, the velocity profiles of Michael are rather 'triangular', whereas those of Gregory are more 'rectangular'.

Another interesting feature of all these signals is the initial rising phase. The time evolution of the height is shown on a log–log scale in figure 5 for Michael (*a*) and Gregory (*b*) and different mass. In all the cases, this presentation reveals that the height initially increases as the third power of time: $z_M \sim t^3$, which implies that the velocity is initially quadratic in time.

4. Model

(a) Joint equation

We model the weightlifter by a single joint as presented in figure 6. The elbow of radius r and mass M_a is connected to two massless bones of identical length L. The shoulder is motionless and defines the origin z = 0. During a single lift, the weightlifter extends his arms by contracting his muscles (red). The barbell goes from $z_M = 0$, with arms bent ($\psi = \pi/2$) to the vertical position $z_M = 2L$ ($\psi = 0$). During the motion, the barbell is in the weightlifter's hands at the location $z_M = 2L \cos \psi$. The muscles are contracting at a velocity $v = -r\dot{\theta} = -r\dot{\psi}$ and generate the force F which enables the lift.



Figure 5. Time evolution of the barbell height in the initial rising phase for Michael (a) and Gregory (b) and for different mass. (Online version in colour.)



Figure 6. Scheme of the elbow joint and associated forces: the arm bones (length *L*) are linked by the joint (radius *r*) and make an angle ψ with the vertical line. The muscle drawn in red, enables to extend the arm and lift the weight *M*. (Online version in colour.)

To derive the equation of motion $\psi(t)$, we first apply Newton's equation in z_M . Along the vertical direction z, this equation writes $M\ddot{z}_M = C_1 \cos \psi - F \cos \theta - Mg$, where C_1 is the force exerted by the bone on the mass. This equation gives an expression for C_1

$$C_1 \cos \psi = M(\ddot{z}_M + g) + F \cos \theta. \tag{4.1}$$

At the elbow, *B*, the equation of motion along the direction *x* takes the form: $M_a \ddot{x}_B = 2F \sin \theta - C_2 \sin \psi - C_1 \sin \psi$. In this equation, C_2 stands for the force exerted by the lower bone on the joint, the expression of which can be deduced

Finally, along the vertical direction, the equation of motion of the joint writes: $M_a \ddot{z}_B = (C_2 - C_1) \cos \psi - M_a g$. Using equation (4.1) for C_1 and equation (4.2) for C_2 , we get

$$(M_a + 4M\sin^2\psi)L\ddot{\psi} + 4ML\sin\psi\cos\psi\dot{\psi}^2 = (M_a + 2M)g\sin\psi - 2F\sin\alpha.$$
(4.3)

To derive this equation, we have used the geometrical relations $\ddot{x}_B = -L \cos \psi \ddot{\psi} + L \sin \psi \dot{\psi}^2$, $\ddot{z}_B = -L \sin \psi \ddot{\psi} - L \cos \psi \dot{\psi}^2$ and $\ddot{z}_M = -2L \sin \psi \ddot{\psi} - 2L \cos \psi \dot{\psi}^2$.

Using $T = \sqrt{L/g}$ as the characteristic time, equation (4.3) can be rewritten under the dimensionless form

$$\left(1+4\frac{M}{M_a}\sin^2\psi\right)\bar{\psi}+4\frac{M}{M_a}\sin\psi\cos\psi\,\bar{\psi}^2 = \left(1+2\frac{M}{M_a}\right)\sin\psi-\frac{2F\sin\alpha}{M_ag}.$$
(4.4)

Equation (4.4) must be solved with the initial conditions: $\psi(0) = \pi/2$ and $\dot{\psi}(0) = 0$. For the mass to rise, the force must be large enough to get $\ddot{\psi} < 0$. This condition implies that if F_0 is the maximal force that the athlete can develop, the maximal mass that he can lift is

$$M^{\star} = \frac{F_0}{g} \frac{r}{L} - \frac{M_a}{2}.$$
(4.5)

For a given force and athlete mass, M^* is maximized by short segments and large articulations. Elongated and thin bodies are thus disadvantaged in weightlifting.

(b) Force relation

At this point, it must be underlined that the differential equation (4.4) can be integrated only if the force *F* is known. Determining this closure relation is the purpose of this section. Using F_0 , the closure term can be written as

$$\frac{2F\sin\alpha}{M_ag} = \frac{F}{F_0} \left(1 + 2\frac{M^\star}{M_a} \right) = \bar{F} \left(1 + 2\frac{M^\star}{M_a} \right), \tag{4.6}$$

where $\bar{F} = F/F_0$ is the reduced force. The mass of the lifting 'machine' M_a can be evaluated using classical physiological studies as the sum $M_a = M_{arm} + M_{forearm}$ where the mass of the two arms is $M_{arm} = 0.08 \times M_0 - 1.31$ and the mass of the two forearms is $M_{forearm} = 0.04 \times M_0 - 0.23$ [19]. For Gregory, we find $M_a = 12.1$ kg and deduce $M^*/M_a = 19$. For Michael, we find $M_a = 7$ kg and deduce $M^*/M_a = 15$.

(i) Constant force: $\overline{F} = 1$

We first naively consider the muscle as an ideal force generator, producing always the maximal force $F = F_0$, whatever the load or the velocity of contraction. We then integrate numerically equation (4.4) with $M^*/M_a = 15$ and present the dynamics of weightlifting in figure 7. Figure 7 shows the results obtained for the mass height (*a*) and mass velocity (*b*) for different masses of the barbell ranging from $M/M^* = 0.1$ to $M/M^* = 0.9$.

Several features differ from the experimental observations presented in figure 4: the initial rise of the velocity is linear here, whereas it seems quadratic experimentally. At low masses, the experimental velocity is almost symmetric, whereas it is always dissymmetric in the numeric. Finally, no secondary peak appears at large mass in the numeric. We conclude that the experimental results cannot be approached with a constant force model. A deeper understanding of muscle contraction must be introduced in order to improve our model.

(ii) Presentation of the sliding filament model

A scheme of the contractile machinery is presented in figure 8a: muscle cells extend from one tendon to another, which connect to bones. The muscle cell is composed of nuclei and of myofibrils, a linear assembly of sarcomeres, the elementary contractile unit. A sarcomere itself is made of thin actin filaments connected to thick myosin filaments via myosin heads (figure 8b(i)(ii)). When a neuron stimulates a muscle cell, an action potential sweeps over the



Figure 7. Evolution of the dimensionless height $z_M/2L$ (*a*) and of the dimensionless vertical velocity $\dot{z}_M(t)/\sqrt{4gL}$ (*b*) as a function of the reduced time $t\sqrt{g/L}$ obtained by integrations of equation (4.4) for different masses of the barbell and with the initial conditions: $\psi(0) = \pi/2$ and $\dot{\psi}(0) = 0$. (Online version in colour.)



Figure 8. (*a*) Structure of the contractile unit from the muscle to the sarcomere. (*b*(i)) Relaxed state of the sarcomere [20]. (*b*(ii)) Contracted state of the sarcomere [20]. (*c*) Motility cycle of muscle myosin–actin interaction. The cycle is composed of three phases: attachment A, force generation B and detachment C. (Online version in colour.)

plasma membrane of the muscle cell. The action potential releases internal stores of calcium that flows through the muscle cell. Actin and myosin filaments are juxtaposed but cannot interact in the absence of calcium (relaxed state b(i)). With calcium, the myosin-binding site is open on the actin filaments, which allows the myosin motors to crawl along the actin, resulting in a contraction of the muscle fibre b(ii) [12,21].

A theory for the contraction based on this sliding-filament model was then proposed by Huxley in 1957 [14] and further completed in 1968 by Deshcherevskii [15]. We present the main steps of this later model and then establish the connection with weightlifting.

Deshcherevskii considers three main states in the actomyosin cycle (figure 8*c*): a myosin head is either 'free (A), or developing an active force (B) or detaching (with a breaking force) (C). Each

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myosin head within a sarcomere moves from one state to the other following the sequence A–B– C–A–B–C–... Denoting *n* and *m* as the number of myosin heads in the states (B) and (C) and α_D the total number of myosin heads in half a sarcomere, Deshcherevskii expresses the rate of change of both populations via two kinetic equations

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\alpha_D - (n+m)}{\tau_1} - \frac{v_{\rm s}}{\ell}n\tag{4.7}$$

and

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{v_{\mathrm{s}}}{\ell}n - \frac{m}{\tau_2},\tag{4.8}$$

where τ_1 is the characteristic time of the transition from state A to B and τ_2 is the characteristic opening time of bridges which leads to a transition from C to A; ℓ is the mean value of conformational transformation of the myosin head during the power stroke ($\ell \approx 10$ nm). If v_s is the relative velocity between the actin and myosin filaments, ℓ/v_s represents the time after which one active head switches from stretched (active) to compressed (resisting). In the isometric limit where there is no net contraction of the sarcomere, the characteristic time of the transition B–C reduces to the characteristic time τ_{nm} of the topological switch of the myosin molecule. One could thus write the velocity $v_s = v_t + v_i$ as the sum of a translation velocity v_t and intrinsic velocity $v_i = \ell/\tau_{nm}$. This later term becomes dominant only in the isometric limit. Because we focus here on the dynamic, this term is going to be neglected in the following discussion where we assume $v_s \approx v_t$.

Assuming that the active and braking force developed by the myosin head are identical and equal to f^1 , the force developed by a sarcomere is equal to

$$F_{\rm s} = f(n-m). \tag{4.9}$$

To extend this result to the whole muscle, we first evaluate the number of sarcomeres aligned in series, N_s . If $L \approx 0.3$ m is the length of the muscle and $\ell_s \approx 3 \,\mu$ m the length of a sarcomere, then one gets $N_{\rm s} \approx 10^5$. This number connects the sliding velocity $v_{\rm s}$ and the contraction velocity v via the relation: $v = N_s v_s$. From Wilkie [6], we get that, in humans, the contraction velocity is in the range $v \approx 1-10 \,\mathrm{m \, s^{-1}}$. This implies that the sliding velocity in the sarcomere is in the range $v_{\rm s} \approx$ 10–100 μ m s⁻¹. The next step consists of evaluating the number of sarcomeres placed in parallel in a muscle, N_p . If S ($S \approx \pi (0.1)^2/4 \approx 10^{-2} \text{ m}^2$) stands for the cross section of the muscle and S_s ($S_s \approx$ $\pi (0.5 \times 10^{-6})^2/4 \approx 2.5 \times 10^{-13} \text{ m}^2)$ for the cross section of a sarcomere, one gets $N_p \approx 4 \times 10^{10}$. The total force of the muscle F is then related to the force developed by a single sarcomere F_s via the relation: $F = N_p \cdot F_s$. Because $F \approx 500$ N, one deduces the number of simultaneously active myosin heads $DC.\alpha_D \approx F/f.N_p \approx 3000$. In this expression, DC is the fraction of active to the total number of myosin heads which can be evaluated as the duty factor which is of the order of 0.2 [26,27]. Which leads to $\alpha_D \approx 15\,000$. This global evaluation can be compared with a direct counting of the number of myosin heads: using the work of Huxley [28], one evaluates eight myosin heads every 43 nm of myosin filament. Because the distance between the thick myosin filament is 50 nm, we deduce that in a sarcomere (500 nm in diameter), the total number of heads is $\alpha_D \approx (500/50)^2 \times$ $(1000/43) \times 8 \approx 18400$. Both evaluations are compatible.

(iii) The steady limit and Hill's equation

In the steady regime where v_s is a constant, the populations of the three stages A,B,C remain constant on average, and one deduces $\tau_1 v_s n/\ell = \alpha_D - (n + m)$ and $\tau_2 v_s n/\ell = m$.

These equations enable us to get the force-velocity relationship

$$F = F_0 \frac{1 - v/v_{\text{max}}}{1 + (F_0/a).v/v_{\text{max}}} \quad \text{with } F_0 = N_p.\alpha_D.f, \quad \frac{F_0}{a} = 1 + \frac{\tau_1}{\tau_2} \quad \text{and} \quad v_{\text{max}} = \frac{N_s.\ell}{\tau_2}.$$
(4.10)

¹*f* has been measured by several groups, and it is found to be of the order of 3 pN [22–25].



Figure 9. Evolution of the dimensionless height $z_M/2L$ (*a*) and of the dimensionless vertical velocity $\dot{z}_M(t)/\sqrt{4gL}$ (*b*) as a function of the reduced time $t\sqrt{g/L}$ obtained by integrations of equation (4.4) and (4.11) for different masses of the barbell and with the initial conditions: $\psi(0) = \pi/2$ and $\dot{\psi}(0) = 0$. (Online version in colour.)

This hyperbolic relation between the force and the velocity was first proposed heuristically by Hill in 1938 [5]. Here, it appears as the quasi-steady limit of the Deshcherevskii's kinetic model. Using the previous estimates, we get $\tau_2 = N_s \ell / v_{max} \approx 10^5 \times 10^{-8} / 1 \approx 1$ ms. What is important to note at this stage is the fact that the maximal velocity of relative motion in the sarcomere is ℓ / τ_2 . The velocity ℓ / τ_{nm} is thus lower, and we deduce $\tau_{nm} > \tau_2$. Moreover, from Wilkie [6], we find $F_0/a \approx 5$ in humans which implies $\tau_1 \approx 4 \tau_2$. These orders of magnitude are compatible with those proposed by Deshcherevskii [29], who evaluate $\tau_1 \approx 3 - 6 \tau_2$ and $\tau_2 \approx 6$ ms.

Because the contraction velocity v is linked to the angle of the joint ψ through the relation $v = -r\dot{\psi}$, equation (4.10) can be rewritten in the form

$$\bar{F} = \frac{1 + G\bar{\psi}}{1 - G\bar{\psi}(1 + \tau_1/\tau_2)}, \quad \text{where } G = \frac{r}{v_{\text{max}}}\sqrt{\frac{g}{L}}.$$
 (4.11)

In the limit $G \ll 1$, this expression of the force reduces to the constant limit $\overline{F} = 1$ which has been discussed in §4b(i). The numerical integration of the system (4.4) and (4.11) with G = 0.3 and $\tau_1/\tau_2 = 3$ is shown in figure 9 for both the height (*a*) and the rising velocity (*b*).

Even if the numerical integrations seem to get closer to the experiments, the initial phase is much too quick and exhibits a behaviour $\bar{z}_M \sim \bar{t}^2$ which is not observed experimentally. We thus now consider the non-steady limit of the full Deshcherevskii's model.

(iv) The unsteady limit of Descherevskii's model

In a non-dimensional form, the kinetic model of Deshcherevskii (4.7), (4.8) and (4.9) takes the form

$$\frac{d\bar{n}}{d\bar{t}} = \frac{(1 - \bar{m} - \bar{n})}{\bar{\tau}_1} - \frac{\tilde{v}\bar{n}}{\bar{\tau}_2}$$
(4.12)

$$\frac{\mathrm{d}\bar{m}}{\mathrm{d}\bar{t}} = \frac{\tilde{v}\bar{n}}{\bar{v}_2} - \frac{\bar{m}}{\bar{v}_2} \tag{4.13}$$

$$\bar{F} = \bar{n} - \bar{m}.\tag{4.14}$$

and



Figure 10. Evolution of the fraction of active myosin heads \bar{n} (*a*), to be detached myosin heads \bar{m} (*b*) and muscle force \bar{F} (*c*). These evolutions result from the numerical integration of equations (4.4) and (4.16), (4.15) and (4.14) for different masses of the barbell and with the initial conditions: ψ (0) = $\pi/2$ and $\dot{\psi}$ (0) = 0, G = 0.3, $\bar{\tau}_2$ = 0.25 and $\bar{\tau}_1$ = $3\bar{\tau}_2$, M^*/M_a = 15. (Online version in colour.)

In these expressions, $\bar{t} = t\sqrt{g/L}$, $\bar{n} = n/\alpha_D$, $\bar{m} = m/\alpha_D$, $\bar{\tau}_1 = \tau_1\sqrt{g/L}$, $\bar{\tau}_2 = \tau_2\sqrt{g/L}$, $\tilde{v} = v/v_{max} = -G\bar{\psi}$ and $\bar{F} = F/F_0$. Equation (4.12) provides the expression $\bar{m}(\bar{n}, \dot{\bar{n}})$:

$$\bar{m} = 1 - \left(1 - \frac{\bar{\tau}_1}{\bar{\tau}_2 G \bar{\psi}}\right) \bar{n} - \bar{\tau}_1 \dot{\bar{n}},\tag{4.15}$$

which can be differentiated with respect to time and injected in equation (4.13) to get the differential equation for \bar{n} :

$$\bar{\tau}_1 \bar{\tau}_2 \ddot{\bar{n}} + (\bar{\tau}_1 + \bar{\tau}_2 - \bar{\tau}_1 G \bar{\psi}) \dot{\bar{n}} + \left[1 - \left(1 + \frac{\bar{\tau}_1}{\bar{\tau}_2} \right) G \bar{\psi} - \bar{\tau}_1 G \bar{\bar{\psi}} \right] \bar{n} = 1.$$
(4.16)

Equation (4.16) must be solved with initial conditions that are going to be discussed below. Once integrated, one gets the time evolution of \bar{n} and deduces via equation (4.15) the value of \bar{m} . Both expressions provide the evaluation of the force $\bar{F} = \bar{n} - \bar{m}$ which can then be used as a closure in equation (4.4), because $2F \sin \alpha/M_a g = \bar{F}(1 + 2M^*/M_a)$. In the quasi-steady limit $(\bar{\tau}_1 \rightarrow 0, \bar{\tau}_2 \rightarrow 0, \bar{\tau}_1/\bar{\tau}_2 \text{ finite})$, equation (4.16) reduces to $\bar{n} = 1/[1 - (1 + \bar{\tau}_1/\bar{\tau}_2)G\bar{\psi}]$ from which $\bar{m} = 1 - (1 - \bar{\tau}_1/\bar{\tau}_2G\bar{\psi})\bar{n}$. These relations together with $\bar{F} = \bar{n} - \bar{m}$ allow to recover Hill's relation $\bar{F} = (1 + G\bar{\psi})/[1 - (1 + \bar{\tau}_1/\bar{\tau}_2)G\bar{\psi}]$.

We now discuss the initial conditions used to integrate equation (4.16). Initially, the athlete holds the weight in the position $\psi = \pi/2$. The weight neither falls nor rises ($\bar{\psi} = 0$). From equation (4.4), we deduce $\bar{F} = (1 + 2M/M_a)/(1 + 2M^*/M_a)$ and $\bar{\psi} = 0$. According to the model of Deshcherevskii presented in figure 8*c* and described by equations (4.7)–(4.9), the population *m* is fed by the population *n* which has produced the force *f* during the time ℓ/v_s and is depleted over the characteristic time τ_2 . In other words, if all the active myosin heads (state B) are initially active, then the population *m* reduces to 0. This is the condition with which we start our calculation: $\bar{m} = 0$ and thus $\bar{n} = \bar{F}$ (equation (4.14)). It follows from equation (4.15) that $\dot{\bar{n}} = (1 - \bar{n})/\bar{\tau_1}$. This condition, together with $\bar{n} = \bar{F}$ provides the two initial conditions needed to integrate the second-order ordinary differential equation (4.16). Apart from the ratio M^*/M_a which has already been discussed in §4b, we take the values of Deshcherevskii: $\tau_2 = 6 \text{ ms}$ ($\bar{\tau}_2 = 0.04$) and $\tau_1 = 6\tau_2$. The only parameter is *G*, and we chose it constant for a given athlete in order to approach the velocity signal at all mass.

An example of numerical integration of equations (4.16), (4.15) and (4.14) with the parameters G = 0.17, $\bar{\tau}_2 = 0.04$ and $\bar{\tau}_1 = 6\bar{\tau}_2$ is presented in figure 10. The evolution of the fraction of active myosin heads \bar{n} is strongly nonlinear and leads to a nonlinear evolution of the muscle force \bar{F} . The corresponding evolution of the barbell height $z_M/2L$ and barbell speed $\dot{z}_M/\sqrt{4gL}$ are obtained with the associated integration of equation (4.4) and presented in figure 11.



Figure 11. Evolution of the dimensionless height $z_M/2L$ (*a*) and of the dimensionless vertical velocity $\dot{z}_M(t)/\sqrt{2gL}$ (*c*) as a function of the reduced time $t\sqrt{g/2L}$ obtained by integrations of equation (4.4) and (4.16), (4.15) and (4.14) for different masses of the barbell and with the initial conditions: $\psi(0) = \pi/2$ and $\dot{\psi}(0) = 0$, G = 0.17, $\bar{\tau}_2 = 0.04$ and $\bar{\tau}_1 = 6\bar{\tau}_2$, $M^*/M_a = 15$. For comparison, the reduced experimental results for Michael are shown in (*b*). (Online version in colour.)



Figure 12. Evolution of the dimensionless height $z_M/2L$ (*a*) and of the dimensionless vertical velocity $\dot{z}_M(t)/\sqrt{2gL}$ (*c*) as a function of the reduced time $t\sqrt{g/2L}$ obtained by integrations of equation (4.4) and (4.16), (4.15) and (4.14) for different masses of the barbell and with the initial conditions: $\psi(0) = \pi/2$ and $\dot{\psi}(0) = 0$, G = 0.33, $\bar{\tau}_2 = 0.04$ and $\bar{\tau}_1 = 6\bar{\tau}_2$, $M^*/M_a = 19$. For comparison, the reduced experimental results for Greg are shown in (*b*). (Online version in colour.)

We first observe in figure 11*a* that the barbell height increases as t^3 as shown experimentally in figure 5. Concerning the velocity, the non-dimensionalized experimental results associated with Michael (figure 4) are presented in figure 11*b* and can be compared with the numerical results presented in figure 11*c*. For the same range of lifted masses M/M^* , the rising time is similar, and the range of velocity seems to differ by a factor 1.3. The main difference between the two sets of curves seems to be in the final fall of the velocity which is systematically sharper in the numerics than in the experiments. This effect is probably due to the fact that both hand and shoulder positions are assumed to be fixed in the numerics while they slightly change in the experiments.

Using the characteristics of the other athlete Greg ($M^*/M_a = 19$) and similar values of the muscle properties (G = 0.33, $\bar{\tau}_2 = 0.04$ and $\bar{\tau}_1 = 6\bar{\tau}_2$), we present in figure 12 the comparison between the reduced experimental results (*b*) and the numerical integration of the joint–muscle system (equations (4.4), (4.16), (4.15) and (4.14)). Again, the initial rise is cubic in time and the numerical signal evolves towards a more rectangular shape, as does the experimental one. The characteristic time for the rise are similar, whereas the absolute value of the velocity still differs by a factor 1.4.



Figure 13. (*a*) Evolution of the lifting dynamics observed numerically in the phase space ($\bar{\tau}_1/\bar{\tau}_2$, *G*) for a constant $\bar{\tau}_2 = 0.04$ and changing *G* and $\bar{\tau}_1/\bar{\tau}_2$. (*b*) Bench press experiments performed by Loïc Auvray, who develops an amylotrophic lateral sclerosis muscle pathology. (Online version in colour.)

5. Discussion and medical perspectives

We have just shown that the microscopic understanding of muscle contraction is essential to account for the motions observed in weightlifting. The model we propose to describe bench press lifting is, indeed, composed of two different parts, one for the articulation (equation (4.4)) and one for the force generation (equations (4.12)–(4.14)). This model involves three different constants, two for the ratio between the biochemical kinetics and the macroscopic timescale ($\bar{\tau}_1$, $\bar{\tau}_2$) and one to compare the characteristic time of muscle contraction with the characteristic time of the macroscopic motion (*G*). Up to now, the first two constants have been fixed to the values estimated in Deshcherevskii's studies. Only *G* was slightly changed in the simulations presented in figures 11 and 12 from 0.17 to 0.33. If one explores numerically, then the whole phase diagram (G, $\bar{\tau}_1/\bar{\tau}_2$) different behaviours appear as shown in figure 13. For low values of $\bar{\tau}_1/\bar{\tau}_2$, the increase of *G* leads to the transition already observed from a triangular behaviour of the velocity in time to a rectangular one. At larger values of $\bar{\tau}_1/\bar{\tau}_2$, oscillations occur either over the whole lift (small *G*), or concentrated in the first phase of the rise (large *G*). This suggests that bench press experiments could be used as a muscle rheometer to probe characteristic times of muscle contraction or to identify muscle pathologies.

To check these ideas, Loïc Auvray has agreed to work with us and performed the same bench press experiments as the other athletes. Loïc developed, eight months ago, an amyotrophic lateral sclerosis. This muscle pathology is a rapidly progressive neurological disease that attacks the nerve cells (neurons) responsible for controlling voluntary muscles. The experimental results obtained with Loïc are presented in figure 13*b*. There, the velocity oscillations are clearly visible and contrast with the smooth evolutions observed in figure 4.

These different behaviours suggest that the analysis of bench press lifts can indeed be used as a muscle rheometer, able to distinguish regular behaviours from pathological ones. This perspective of using bench press lifts as a diagnostic tool is a very promising and stimulating perspective of this work.

Ethics. Most of the data presented in the article have been obtained in a sports centre with informed consent members. The data presented in figure 13*b* were obtained with Loïc Auvray who is an author of the article.

Data accessibility. The experimental data presented are accessible upon request to the authors.

Authors' contributions. C.C. carried out the experiments and participated in both the data analysis and the elaboration of the model; B.D.T. was involved in the elaboration of the model, G.L. selected the athletes, carried out the experiments and participated in the data analysis. L.A. performed experiments and participated in the discussion on the model; C.C. analysed the data, developed the model and drafted the manuscript. All authors gave final approval for publication.

Competing interests. We have no competing interests.

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