Froude number dependence of the flow separation line on a sphere towed in a stratified fluid

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In this paper experimental results on the near field of the flow past a sphere in a linearly stratified medium are presented. Emphasis is placed on the variation of the flow separation line with internal Froude number $F = U/NR$ and also with Reynolds number $\mathrm{Re} = 2RU/\nu$, where U and R are, respectively, the velocity and the radius of the sphere, N is the Brünt–Väisälä frequency (rad sec^{-1}), and ν is the cinematic viscosity. It is shown that in the Reynolds number range $200 < \mathrm{Re} < 30\,000$ the flow is primarily conditioned by the Froude number when $F \leqslant 1$. The condition $F = 1$ defines a resonance state between the sphere and the internal wave field. In this case the waves create a strong depression behind the sphere that keeps the flow from separating. When $F < 0.8$ the flow is two dimensional in a layer confined between the upper and the lower wave. When $F > 1.5$ the flow starts to recover its three-dimensionality.

I. INTRODUCTION

The flow of stratified fluid past obstacles representing hills or mountains is of geophysical interest (Miles[1]). Depending on flow conditions, the flow downstream of the obstacle can stay nearly attached and give rise to strong internal waves (lee waves), or separate and give rise to a turbulent wake with a recirculating zone in the lee of the mountains. Brighton[2] showed the tendency of three-dimensional stratified flows past obstacles for small Froude numbers $(0.03 < F < 0.3)$ to be confined to horizontal planes. Hunt and Snyder[3] studied experimentally the stratified flow past surface mounted obstacles, representing a three-dimensional hill, and demonstrated the correlation between flow separation and lee waves.

The stratified flow past free obstacles is a related problem with the surface conditions replaced by a plane of symmetry. The flow past a sphere at low Reynolds number was studied numerically by Hanazaki.[4] Sysoeva and Chashechkin[5] and Chashechkin and Sysoeva[6] reported experimental results on flow separation on a sphere and the near wake structure in stratified fluid for Reynolds numbers ranging from 24 to 1000 and Froude numbers in the range $0.3 < F < 7$. In these experiments, the flow separation angles θ, measured from the upstream stagnation point, differ from the numerical results of Hanazaki.[4] In the numerical simulations, done for a Reynolds number $\mathrm{Re} = 200$, it was found that flow separation is completely suppressed, i.e., $\theta = 180°$ in the vertical and horizontal median planes, when $F = 1$. For the same Reynolds number, Sysoeva and Chashechkin[5] obtain θ never greater than 165°. These authors used a shadowgraph technique to determine the separation angle. This technique integrates over the whole depth of the wake and this can lead to erroneous interpretation. Chashechkin and Sysoeva[6] also used an electrolytic precipitation method to visualize the whole flow separation line. However, in their study the Froude number seemed to have been $F \leqslant 0.4$ and, because of this restriction to small Froude numbers, only a square pattern of the separation line was found.

Here we present results about flow separation on a sphere moving in a stratified fluid and the associated internal wave field for $0.3 \leqslant F \leqslant 7$ and $200 \leqslant \mathrm{Re} \leqslant 30\,000$. It is shown that for low Reynolds numbers the numerical results of Hanazaki[4] are supported by experiments. Flow visualizations give also an indication of the topology of the separation line and the wake structure.

II. EXPERIMENTAL APPARATUS

The experiments were conducted in two tanks: a glass tank 50 cm wide, 50 cm deep, and 4 m long and a very large tank 3 m wide, 1 m deep, and 20 m long, with incorporated glass walls. Three different spheres were used having radius $R = 1.1, 2.5$, and 3.6 cm. These were ballasted with lead and suspended from a frame by three steel wires 0.1 mm thick (Chomaz *et al.*[7]). A linear salt stratification with N $[N = (-g/\rho_0\, d\rho/dz)^{1/2}]$ between 1.26 and 2.02 rad sec^{-1} was established by the two-tank filling technique or by a computerized process. The towing speeds ranged between 0.8 and 50 cm sec^{-1} giving Reynolds numbers of 200–30 000.

The ratio of sphere radius/half-depth of the channel (R/D) is small in the present experiments, ranging from $R/D = 0.022$–0.144. Confinement effects on the flow structure are therefore likely to be negligible. The conditions for confinement to be negligible in stratified flows were discussed in Hopfinger *et al.*[8]

We used three different techniques to visualize the flow: the shadowgraph technique, a fluorescent dye technique, and the particle streak-line trajectories. In the two last techniques, horizontal or vertical planes were illuminated by a

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laser sheet, whereas in the shadowgraph technique, the whole variation of the density along a light ray affects the image, giving information about the three-dimensional variation of the flow structure. In the case of the fluorescence induced by laser light, the fluorescent dye was deposited on a zone of the sphere surface variable at will in location and size. The dye was deposited in different experiments on forward and backward facing zones, visualizing, respectively, the detached flow and the recirculation zone. The laser light sheet illumination allowed an accurate determination of the separation points in the vertical and horizontal planes. The topography of the separation lines was obtained by illuminating the whole near field of the wake by means of a mercury vapor spotlight and by observating the flow at about 45° to the displacement direction of the sphere. The photographs taken in this way, do not, unfortunately reproduce the perspective view of the human eye.

III. EXPERIMENTAL RESULTS

As a first step, the wake structure in homogeneous fluid was investigated because this knowledge is essential to the interpretation of the stratification effect (Bonneton *et al.*[9]). Results concerning the position of separation (Pruppacher *et al.*[10]) give an angle θ, measured from the front stagnation point, which decreases from 180° to about 80°, in the range $\mathrm{Re} \in [25, 500]$. The range $\mathrm{Re} \in [500, 300\,000]$ defines the regime corresponding to a laminar boundary layer, where the neutral separation line is fixed at 80°. When Re is greater than 300 000, the boundary layer starts to become turbulent and the angle increases abruptly to 120°. By means of the fluorescent dye technique, the same variation of the separation angle with Re is obtained, except that we observe a saturation of θ at an angle of 90°. This is because, for θ smaller than 90°, flow separation occurs nearly tangent to the sphere and visualizations give a poor estimation. For θ greater than 90°, flow separation is at a nonzero angle with respect to the sphere surface and our optical measurements give good accuracy.

In the presence of stratification the experiments were made in the laminar boundary-layer regime. When, for a given stratification and a given sphere, the velocity is varied, the two dimensionless numbers F and Re evolve proportionally for each data set in the form $\mathrm{Re}(F) = \mathrm{Re}(1)F$, where $\mathrm{Re}(1) = 2R^2N/\nu$. Experiments were performed for Re(1) equal to 324, 1961, and 4065. Figures 1(a)–1(c) show, for $F = 0.8$ and $\mathrm{Re}(1) = 1961$, side views obtained by the three different visualization techniques. The flow separation point is indicated by an arrow in these figures. The shadowgraph image [Fig. 1(a)], which is sensitive to the whole density field along light rays, reveals two separation points marked by arrows. In fact, the lower one is in the median vertical plane, whereas the upper one corresponds to the maximum height of the separation curve, which occurs off the median vertical plane. The fluorescent dye technique [Fig. 1(b)] allows a more precise and unambiguous determination of the separation angle in the vertical median plan, because this plan is lighted by a thin laser light sheet. The particle streak image [Fig. 1(c)] provides a coarse estimate of the separation line because the probability of finding a particle close to the separation line is very small.

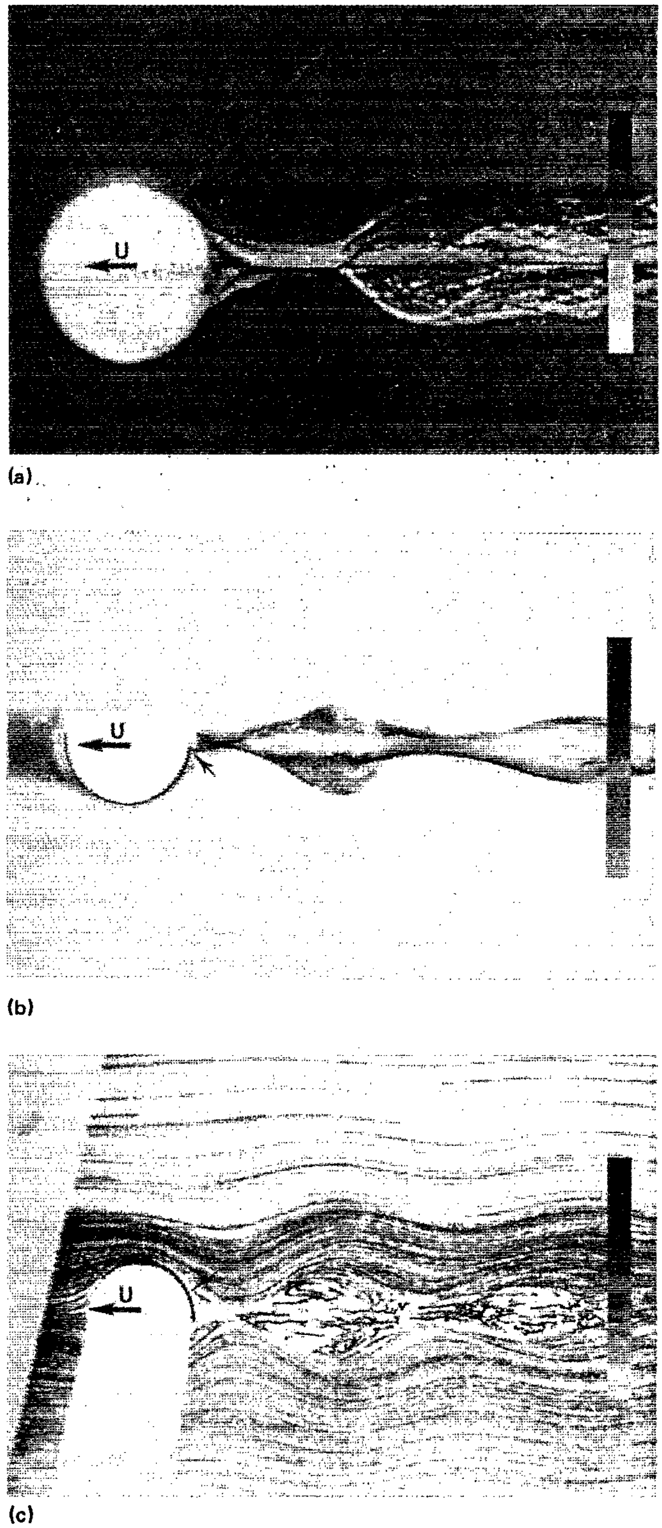


FIG. 1. Side view of the wake behind a sphere moving as indicated by the big arrow in a stratified fluid of $F = 0.8$, $\mathrm{Re} = 1569$ and $R = 2.5$. (a) Shadowgraph; (b) fluorescent technique; (c) particle trajectories; ↙, indicates point of flow separation.

In Figs. 2(a) and 2(b) are presented the separation angles in the vertical plane θ_V, and in the horizontal plane θ_H,

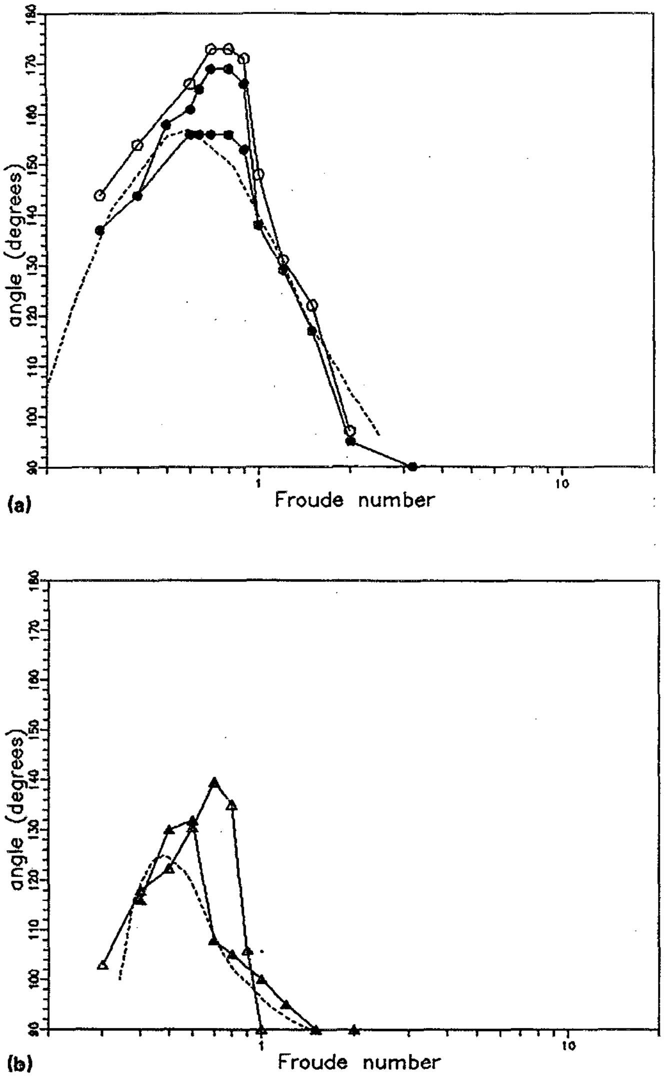


FIG. 2. Variation with F of the separation points on the sphere. Comparison between fluorescent dye technique (○, △) and shadowgraph technique (●, ▲). (a) In the vertical plane for Re(1) = 1961; (b) in the horizontal plane for Re = 4065. - - -, Lofquist and Purtell's[11] results.

determined from fluorescent dye (open symbols) and shadowgraph pictures (solid symbols) as shown in Figs. 1(a) and 1(b). We have included Lofquist and Purtell's[11] results obtained for Re(1) ∈ [1357, 2258] with the shadowgraph technique. This technique reveals on side views for $F \in [0.5, 0.9]$ and Re(1) = 1961 two separation points [Fig. 1(a)]. The upper one (smaller values of θ) is in agreement with Lofquist and Purtell's[11] measurements and the lower one corresponds to the separation point in the vertical median plane. It is seen that the fluorescent dye technique gives values of θ_V close to those obtained with the shadowgraph technique when correctly interpreted. With respect to a mean value, the difference between the two is within $\pm 2°$ for $F > 0.6$ and slightly larger for $F < 0.6$, where the shadowgraph gives poor contrast. In Fig. 2(b) we show that the two techniques give, however, different results for θ_H. In particular, the shadowgraph technique overestimates θ_H for $F > 1$ and gives a bad location for the maximum of θ_H. Moreover, in a horizontal plane, separation induces very small variations of density which lead to a weak contrast of horizontal shadowgraph visualizations. Therefore, we have chosen the fluorescent dye technique because it gives an unambiguous determination of the separation angles θ_V and θ_H in a well-defined plane.

In Figs. 3(a) and 3(b) the separation angles in the vertical median plane θ_V, and in the horizontal median plane θ_H, for three different values Re(1) are presented. In these figures are plotted (in heavy line) Hanazaki's[4] results for Re = 200, a Reynolds number for which the asymptotic value of 80° is not reached. The curves show a complete inhibition of separation around $F = 1$ in both planes for low Reynolds numbers and a considerable decrease in θ for larger values of Re. This is due to the resonance between the sphere

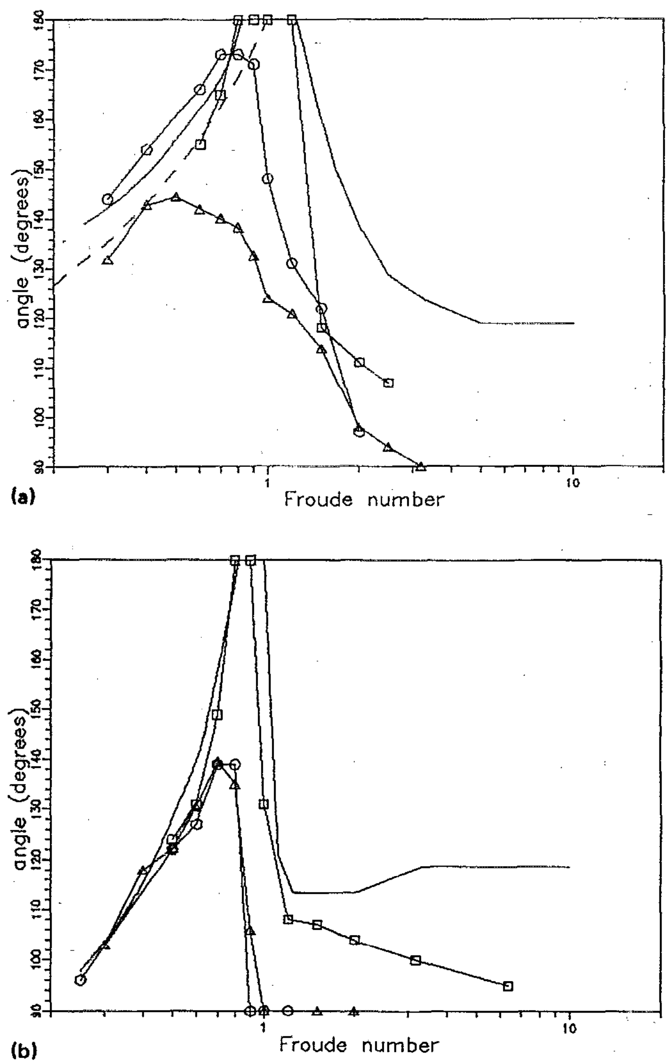
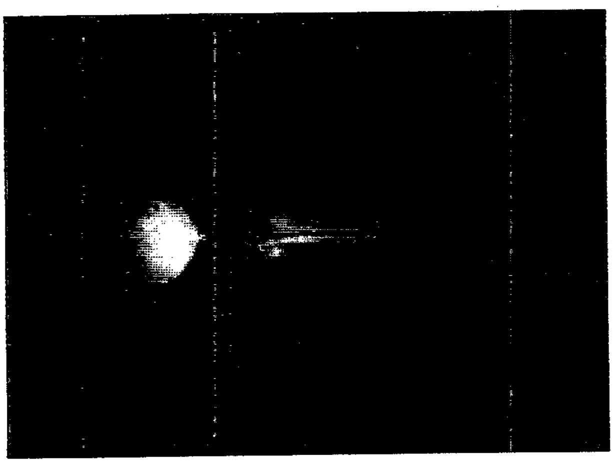


FIG. 3. Variation with F of the separation points on the sphere. (a) In the vertical plane; (b) in the horizontal plane. □, Re(1) = 324; ○, Re(1) = 1961; △, Re(1) = 4065; ——, Hanazaki's numerical simulations; - - -, Sheppard's theory.

and its lee wave which occurs when the sphere size $2R$ equals the half wavelength of the lee wave $\lambda = 2\pi U/N$. Consequently, the amplitude of the lee wave is maximum and the associated depression, just behind the sphere, keeps the flow from separating in both the vertical and horizontal median planes. This is illustrated in Figs. 4(a) [Re(1) = 324] and 4(b) [Re(1) = 1961] corresponding to $F = 0.9$, where the whole wake was visualized with a mercury vapor spotlight. Lofquist and Purtell[11] found that the maximum of θ is reached for $F \simeq 1/\sqrt{2}$. This result corresponds to Re(1) close to 2000, and cannot be extrapolated as suggested by the authors, for other values of Re.

For small Reynolds numbers and $F \simeq 1$, the flow stays attached until the back of the sphere [Fig. 4(a)] and $\theta_V = \theta_H = 180°$ as in Hanazaki's[4] numerical simulations. At higher Reynolds numbers the turbulence of the wake seems to oppose this effect by its associated turbulent pressure term and we observe just behind the sphere a small recirculating zone [Fig. 4(b)]. Therefore, the maximum angle of flow separation decreases with increasing Reynolds number [Figs. 3(a) and 3(b)]. The limiting value of θ at larger than 10^4. Since N cannot be increased it would be necessary to increase the sphere radius but this would lead to possible confinement effects. The variations of θ_V and θ_H with F also depend on Re when $F > 1$. In the vertical plane the separation angle decreases more gradually to the neutral value when Re is large. In the horizontal plane the change remains abrupt also at large Reynolds numbers, and we note the similarity in behavior of θ_H for Re(1) = 1961 and Re(1) = 4065. In general, at large Reynolds numbers the flow recovers its three-dimensionality more rapidly than at low values of Re. In Hanazaki's[4] simulation also, the neutral value of θ_V is almost reached near $F \simeq 3$. At the low Froude number side, $F < 0.8$, gravity effects become dominant making flow separation less sensitive to the Reynolds number. The vertical separation angle is controlled by the lee wave. Following Sheppard's[12] analysis (Snyder *et al.*[13]), based on the Bernouilli equation, we can estimate the Froude number dependence of the separation angle: $\theta_S = 180 - \arcsin(1 - F)$. This value, plotted in a dashed line in Fig. 3(a), agrees with our observations for $F < 1$. However, measurements give slightly larger values of θ than θ_S. Perhaps, this is due to Sheppard's hypothesis which considers that the pressure term in the Bernoulli equation is negligibly small. The horizontal separation angle decreases rapidly with Froude number. The flow goes around the sphere in the horizontal plane and forms a wake of two-dimensional motion.

The difference in behavior between flow separation in the vertical and horizontal planes implies the existence of an unusual shape of the separation line. From a large set of experiments with fluorescein coatings on the sphere surface, we have been able to approximate the shape of the separation line. Four main topologies of the separation line are presented in Figs. 5(a)–5(d) for Re(1) = 1961. The general sequence depends on the Reynolds number, particularly around $F = 0.9$ [Fig. 5(c)]. In this case, three different separation lines are observed for, respectively, Re(1) equal 324, 1961, and 4065, the surface of separated flow being smaller for small Reynolds number as may be deduced from Figs. 3(a) and 3(b). The bow-tie shape for Re(1) = 1961 [Fig.


(a)

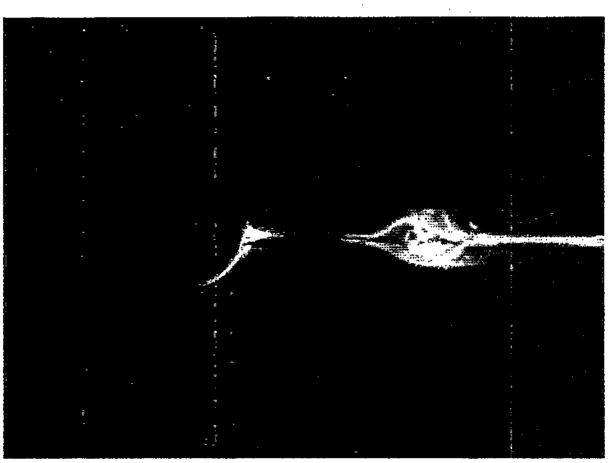

(b)

FIG. 4. Global visualization of the wake with a mercury vapor spotlight for $F = 0.9$. (a) Re(1) = 324; (b) Re(1) = 1961.

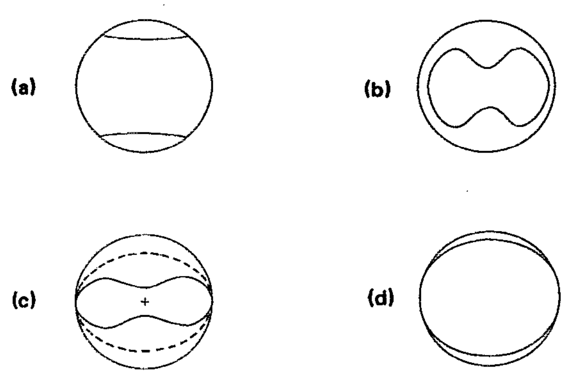
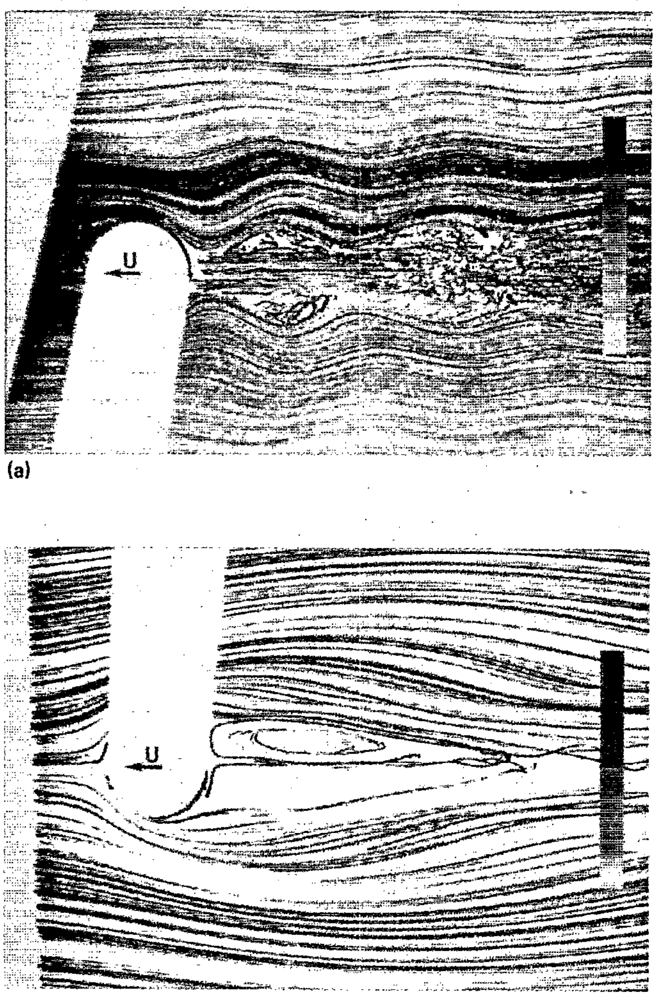


FIG. 5. Shape of the flow separation line on the sphere. (a) $F = 0.3$, Re(1) = 1961; (b) $F = 0.6$, Re(1) = 1961, (c) $F = 0.9$, +, Re(1) = 324, ——, Re(1) = 1961, ——, Re(1) = 4065; (d) $F = 1.5$, Re(1) = 1961.



(a)

(b)

FIG. 6. Particle streak photographs for $F = 0.6$ and $\mathrm{Re} = 1293$. (a) Side view; (b) top view.

5(c)] is characteristic of the resonant case. It reduces to a point in the center [Fig. 4(a)] in the experiment where $R(1) = 324$ in the range $F \in [0.8, 0.9]$ and to a pinched bow-tie shape for $F \in [1, 1.2]$. This is because for $F \in [1, 1.2]$, θ_H is equal to 180 and θ_V is different from 180; two separation bubbles then exist. Figure 5(a) shows the characteristic square shape of the small Froude number regime apparently already observed by Chashechkin and Sysoeva.[6] It results from the partition of the flow into three layers: a two-dimensional layer affected by vertical horizontal motion surrounded by two zones dominated by the lee waves (Fig. 6) (Bonneton *et al.*[14]). The separation induced by the lee waves seems to be not quite along a horizontal line but is slightly depressed in the middle as emphasized in Fig. 5(a). Figure 5(b) shows the shape of the separation line intermediary between the two-dimensional wake and the resonant state. For Froude numbers greater than the resonant state [Fig. 5(d)] the vertical and horizontal symmetry is only slightly broken with a small discrepancy between horizontal and vertical separation angles. At $F = 2$ the separation line is nearly a circle as in the neutral case.

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[1] J. W. Miles, "Waves and wave drag in stratified flows," in *Proceedings of the XII International Congress of Applied Mechanics*, edited by M. Hétenyi and W. G. Vincenti (Springer-Verlag, Berlin, 1969), p 50.

[2] P. W. M. Brighton, "Strongly stratified flow past three-dimensional obstacles," Q. J. R. Meteorol. Soc. **104**, 289 (1978).

[3] J. C. R. Hunt and W. H. Snyder, "Experiments on stably and neutrally stratified flow over a model three-dimensional hill," J. Fluid Mech. **96**, 671 (1980).

[4] H. Hanazaki, "A numerical study of three-dimensional stratified flow past a sphere," J. Fluid Mech. **192**, 393 (1988).

[5] E. Y. Sysoeva and Y. D. Chashechkin, "Vortex structure of a wake behind a sphere in a stratified fluid," J. Appl. Mech. Theor. Phys. (Novosibirsk) 2, 40 (1986).

[6] Y. D. Chashechkin and E. Y. Sysoeva, "Fine structure and symmetry of the wake past a sphere in a stratified liquid," in *Proceedings of 17th Session of B.S.H.C.* (Publisher, Varna, Bulgaria, 1988), Vol. 1, p. 10-1.

[7] J. M. Chomaz, P. Bonneton, A. Butet, E. J. Hopfinger, and M. Perrier, "Gravity wave patterns in the wake of a sphere in a stratified fluid," in *Proceeding of Turbulence 89: Organized Structures and Turbulence in Fluid Mechanics*, edited by M. Lesieur and O. Métais (Kluwer Academic, Grenoble, 1911).

[8] E. J. Hopfinger, J. B. Flör, J. M. Chomaz, and P. Bonneton, "Internal waves generated by a moving sphere and its wake in a stratified fluid," Exp. Fluids **11**, 255 (1991).

[9] P. Bonneton, J. M. Chomaz, and E. J. Hopfinger, "Vortex shedding from spheres at subcritical Reynolds number in homogeneous and stratified fluid," in *Proceedings of Nato Workshop*, edited by S. J. Jimenez (Plenum, New York, in press).

[10] H. R. Pruppacher, B. P. Le Clair, and A. E. Hamielec, "Some relations between drag and flow pattern of viscous flow past a sphere and a cylinder at low and intermediate Reynolds numbers," J. Fluid Mech. **44**, 781 (1970).

[11] K. E. B. Lofquist and L. P. Purtell, "Drag on a sphere moving horizontally through a stratified liquid," J. Fluid Mech. **148**, 271 (1984).

[12] P. A. Sheppard, "Airflow over mountains," Q. J. R. Meteorol. Soc. **82**, 528 (1956).

[13] W. H. Snyder, R. S. Thompson, R. E. Eskridge, R. E. Lawson, I. P. Castro, J. T. Lee, J. C. R. Hunt, and Y. Ojawa, "The structure of strongly stratified flow over hills: Dividing-streamline concept," J. Fluid Mech. **152**, 249 (1985).

[14] P. Bonneton, J. M. Chomaz, and M. Perrier, "Interaction between the internal wave field and the wake emitted behind a moving sphere in a stratified fluid," in *Proceedings of the Conference of Engineering Turbulence Modelling and Experiments*, edited by W. Rodi and E. N. Ganic, (Elsevier, Dubrovnik, Yugoslavia, 1990), p. 459.